MODELS FOR ESTIMATING PLOT SIZE IN EXPERIMENTS

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▪ ABSTRACT: The use of statistical models in the estimation of experimental plot size is a practice that contributes to the experimental planning by choosing a size that allows better efficiency in the comparison of treatments. This work aimed to present some statistical models, with the purpose of using them as alternatives in determining the optimum plot size in experiments. Some nonlinear models with a simple configuration similar to that proposed by the modified maximum curvature technique were proposed, which are derivable and have a curvature function. The curvature function was obtained for each model and plot size estimators were obtained through the critical point of the curvature function derivative. The models were shown to be feasible for estimating plot sizes with simpler estimators compared to those obtained by Meier and Lessman. As an illustration, data from two uniformity tests were used for the application of the proposal and comparison with the modified method of maximum curvature. Estimates of the optimum plot size varied according to model and method.

▪ KEYWORDS: Maximum curvature technique; experimental precision; experimental error; coefficient of variation; uniformity trial.

1 Introduction

In planning an experiment, one must consider the factors that affect the quality of information to be obtained. Among these, the plot size stands out, whose determination, when performed in a judicious way, reduces the effect of environmental variation, provides better precision and efficiency to the experiment and improves the quality of observations collected, making it possible to detect smaller differences among treatments.

Several methods have been used to determine the size of experimental plots. Among the methods used to estimate the optimum plot size, the method of maximum curvature stands out, initially using the variance between plots as the variable that measures the variability of an experiment (SMITH, 1938). Then, the method of maximum curvature

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(MMC) proposed by Lessman and Atikens (1963), determines the optimum plot size by the critical point of the region of maximum curvature that relates the coefficient of variation to its corresponding plot sizes. An improvement of this method was suggested by Meier and Lessman (1971). They obtained the value of the abscissa in which the point of maximum curvature occurs using the following estimator:

$$\hat{X}_c = [\hat{a}^2\,\hat{b}^2\,(2\hat{b}+1)/(\hat{b}+2)]^{1/(2\hat{b}+2)}, \tag{1}$$

where $\hat{X}_c$ is the value of the abscissa at the point of maximum curvature, which corresponds to the estimate of the optimum size of the experimental plot. This model has been shown to be adequate, with consistent results, according to Viana *et al*. (2002) and Silva *et al*. (2003). However, for other authors, MMC underestimates the optimum plot size, as mentioned by Leite *et al*. (2006) and Paranaíba *et al*. (2009).

Segmented regression models have been used to evaluate sample sufficiency and to estimate plot size (GOMIDE *et al*., 2005; COSTA JÚNIOR *et al*., 2008; PARANAÍBA *et al*., 2009; CARGNELUTTI FILHO *et al*., 2011; PEIXOTO *et al*., 2011). They have presenting good fits and being suitable for obtaining optimum plot size closer to those normally used. Barros and Tavares (1995) presented a method that allows obtaining algebraic solution to the point of maximum curvature. Lorentz, Erichsen and Lucio (2012) proposed the method of maximum distance as an alternative proposal to estimate the optimum plot size, and the results were consistent with MMC and linear-plateau model (LPM).

Saste and Sananse (2015) presented a review of the main contributions on plot size estimation, emphasizing the need to minimize the experimental error for the control of soil heterogeneity. The soil heterogeneity index was determined by Saste and Sananse (2016) using the model of Smith (1938) and the semivariogram technique, which considers the spatial dependence among adjacent basic units. Lúcio *et al*. (2016) used the non-linear logistic and von Bertalanffy models to adjust the production of beans in four uniformity trails and estimated the plot size (PS) that provided better model adjustment power.

The method of maximum curvature and relative efficiency were used by Lohmor *et al*. (2017 a, b) to obtain the optimum plot size and the shape of blocks in a uniformity trial with sunflower. The method of maximum curvature of the variation coefficient model, proposed by Paranaíba *et al*. (2009), was used by Lavezo *et al*. (2017) in the estimation of the optimum plot size and number of replications in an experiment to evaluate the production of oat cultivars. Using the percentage efficiency of different plot sizes, Shah *et al*. (2017) found that long and narrow plots were more efficient compared to smaller and wider plots of the same size.

Plot size and number of replications are practical questions related to experimental planning, and their determination and efficient use allow obtaining greater accuracy. The appropriate use of plot size and shape is crucial for different experiments, because what is sought to detect is the existence of significant differences among treatments tested, which depends on the reduction of the experimental error, and can be minimized by the use of plots with optimum size (RAMALHO *et al*., 2012; STORCK *et al*., 2016).

There are several methods for estimating optimum plot size, which can provide varied results when applied in the same set of data, and it is up to the researcher to choose the one that best suits a specific situation. The variability measured by the coefficient of variation (CV) always shows a decrease with the increase in the number of basic units (BU), indicating an improvement in the experimental precision. However, this precision gain becomes non-compensatory by increasing the plot size excessively in numbers of basic units (RAMALHO *et al*., 2012). Since the interest is to obtain X_c value that minimizes the loss of precision, the use of alternative methods can provide results more coherent with the practical situations of each research.

Given the possibility of providing the researcher with some options regarding the size of experimental plots, this work had the aim of presenting some models and their expressions for estimating optimum plot size.

2. Material and methods

Based on some simpler models, but whose behavior of the response variable (y) in relation to the explanatory variable x is always decreasing, as with the coefficient of variation (CV) in relation to the plot size in number of basic units (x), we present some new options for estimating the plot size.

2.1 Proposed models

In order to obtain the models proposed as alternatives for estimating the optimum plot size, an equation that could clearly express the behavior of the coefficient of variation (represented by y or CV) as a function of the plot size (x) and that meets the following characteristics was proposed:

i) y was a strictly decreasing function (should be decreasing);

ii) $\lim_{x \to 0} y = \infty$, where ∞ refers to the highest possible CV;

iii) $\lim_{x \to \infty} y = 0$ refers to the lowest possible CV;

iv) y is a simple, derivable function, being possible to obtain the curvature function and the curvature function has a critical point.

After obtaining the derivatives, the curvature function equation was obtained for each model using the expression suggested by Anton *et al*. (2014):

$$K(x) = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}}, \qquad (2)$$

where y' is the first-order derivative of the function $y = f(x)$ and y'' is the second-order derivative of the function $y = f(x)$. The critical point of the derivative of $K(x)$ provides the estimator for the calculation of the point of maximum curvature, which corresponds to the optimum plot size.

2.2 Mixed algebraic method

This method was proposed in order to take advantage of estimates obtained by the modified method of maximum curvature used by Meier and Lessman (1971), and to relate the derivative of this function to the equation of a straight line taken on two more extreme points. Based on the information obtained from a uniformity trial, considering the pairs of values (x, y), where x represents the number of basic units of a given plot size and y the corresponding value of the coefficient of variation per BU of plots with x BU.

Initially, a line r is plotted, connecting the two most extreme points of the curve (x_1, y_1) and (x_2, y_2), where x_1 is the smallest plot size value and x_2 is the largest, y_1 is the coefficient of variation (CV) related with x_1 and y_2 the CV related to x_2. A line s parallel to line r has the same slope, so the angular coefficients are the same. Since the angular coefficient is a slope at a point (x, y), the line s has the same slope as the line r, given by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}. \qquad (3)$$

The equation of the line by two points is $y - y_0 = m(x - x_0)$ and the slope of the line by two points is the expression of (3), according to Anton et al. (2014). But, this slope can also be obtained by the derivative of the function $y = f(x)$ at point (x, y). Thus, at point (x, y), the derivative of the function y is equal to the slope of the line s, and it is at this point that the line s (tangent line) has its greatest distance from the line r (secant line). Thus, by equating the slope of the line by two points ($(x_1, y_1), (x_2, y_2)$) with the derivative or rate of change of function y, used in the modified method of maximum curvature (MMC), we have for the case of the model

$$y = a / x^b, \qquad (4)$$

where

$$m = \frac{y_2 - y_1}{x_2 - x_1} = -a\,b\,x^{-(b+1)}. \qquad (5)$$

Solving this equation from (5) to x, it was obtained that the estimator is:

$$x_c = \left(\frac{\hat{a}\,\hat{b}\,(x_2 - x_1)}{(y_1 - y_2)} \right)^{\frac{1}{\hat{b}+1}}, \tag{6}$$

which allows estimating the plot size (in number of BU) in a quite simple manner. The same procedure was performed for the other proposed models, obtaining estimators for each model.

2.3 Application

As an illustration of the proposal, results of uniformity trials conducted by Cipriano *et al*. (2012) for estimating plot size for the development of coffee seedlings (Table 1), considering variables dry mass of the shoots (MSPA) and total dry mass (MST), and by Silva *et al*. (2012) for radish crop, considering variables mass and diameter of the tuber.

Table 1 - Estimates of the coefficients of variation (%) for variables dry mass of shoots (MSPA) and total dry mass (MST) of coffee seedlings Rubi MG 1192 cultivar and radish mass and diameter according to different plot sizes X (in numbers of basic units)

Coffee [1]			Radish [2]		
X	MSPA	MST	X	Mass	Diameter
1	35.2	36.16	1	50.9388	32.4736
2	24.8	23.34	2	35.6962	23.2913
3	20.9	20.88	4	26.1456	17.2692
4	17.6	16.75	5	24.9726	15.7650
6	14.3	13.59	8	19.3434	12.8449
8	13.1	11.43	10	17.7697	11.1235
9	10.8	10.27	16	14.8560	10.2014
12	11.0	9.97	20	13.4682	8.2670
16	9.25	7.02	25	13.2542	8.5555
18	7.87	6.59	40	9.9404	5.1552
24	9.12	7.32	50	9.5002	5.1867
36	6.45	5.23	80	8.8092	4.1251
48	7.19	4.72	100	7.2027	2.7888
72	6.69	5.08	200	6.0514	1.2136

[1, 2] Adapted from Cipriano *et al.* (2012) and de Silva *et al.* (2012), respectively.

The optimum plot size was calculated by the modified maximum curvature technique for all proposed models, and also using the expression proposed by Lessman and Atkins (1963), to serve as a comparison. By this method, the relationship between the coefficient of variation (CV) and plot size with X basic units is explained by the model

$$CV = a/X^b + e, \tag{7}$$

where a and b are the parameters to be estimated. From the curvature function given by this model, the value of the abscissa in which the point of maximum curvature occurs, given by

$$\hat{X}_0 = [\hat{a}^2\hat{b}^2(2\hat{b}+1)/(\hat{b}+2)]^{(1/(2+2\hat{b}))}, \tag{8}$$

where $\hat{X}_c$ is the value of the abscissa at the point of maximum curvature, which corresponds to the estimation of the optimum experimental plot size (Meier and Lessman, 1971).

To evaluate the goodness-of-fit of models, the residual standard error (RSD), the adjusted determination coefficient and the Akaike information criterion (AIC) were estimated; lower RSD and AIC values and higher adjusted determination coefficients indicate better fit. Analyses were performed using the free software R (R CORE TEAM, 2017).

3. Results and discussion

3.1 The proposed models and their estimators

The proposed models and their first and second derivative expressions and the expressions obtained for the optimum plot size estimators (x_c) in number of basic units (BU) presented in Table 2 indicate suitable expressions that are easy to use mainly from the practical point of view. Regarding the MMC method, the estimators of the proposed models are simpler, sometimes depending on only one parameter, such as the Mxi, Mrx and Mei models.

Without loss of generality, the following illustrates how the estimators presented in Table 2 were obtained, considering only the case of the Mrx model. For this model, we have the form:

$$y = a + \frac{b}{\sqrt{x}}, \tag{9}$$

with first and second derivatives, respectively, equal to:

$$\frac{\partial y}{\partial x} = \frac{-b}{2\,x^{3/2}} \;\; and \;\; \frac{\partial^2 y}{\partial x^2} = \frac{3b}{4\,x^{5/2}} \tag{10}$$

and whose curvature function (Anton *et al*., 2014) is:

$$K(x) = \frac{\left|3b/(4\,x^{3/2})\right|}{\left\{1 + [-b/(2\,x^{3/2})]^2\right\}^{3/2}} \; = \; \frac{6\,b\,x^2}{\left(4\,x^3 \, + \, b^2\right)^{3/2}}\,. \tag{11}$$

The maximum critical point of the curvature function was obtained by deriving the curvature function in relation to x, obtaining

$$K'(x) = \frac{12\,b^3\,x \, - \, 60\,b\,x^4}{\left(4\,x^3 \, + \, b^2\right)^{3/2}}\,. \tag{12}$$

Equating this expression of (12) to zero and solving at x, the point of maximum curvature (xc) is estimated by

$$x_c \, = \left(\hat{b}^2 \, / \, 5\right)^{1/3}, \tag{13}$$

where $\hat{b}$ is the estimator of parameter b.

Expression (13) is much simpler than that obtained by the method of maximum modified curvature used by Meier and Lessman (1971). All proposed models have curves of the strictly decreasing type, which leads the first derivative to have negative values; with these trends, the models describe the variability, measured by the coefficient of variation, relative to the number of basic units appropriately.

The variability of an experiment is affected by the number of BU that constitute the plot. The plot size directly affects the experimental accuracy as observed, for example, with the coefficient of variation values for the dry matter of shoots, which varied from 35.2% for plot size formed by one basic unit (UB) up to approximately 6.5% with plots formed by more than 36 UB (Table 1).

Table 2 - Expressions obtained for the first and second derivatives of the proposed models, respective estimators for plot size by the method of maximum curvature (x_c) and the algebraic method (x_a)

Model	Derivative		Plot Size	
	1ª	2ª	x_c	x_a
MMC $y = \frac{a}{x^b}$	$\frac{-ab}{x^{b+1}}$	$\frac{ab(b-1)}{x^{b+2}}$	$\left[\frac{\hat{a}^2\hat{b}^2(2\hat{b}+1)}{\hat{b}+2}\right]^{\frac{1}{2\hat{b}+2}}$	$\left(\frac{\hat{a}\hat{b}(x_2 - x_1}{(y_1 - y_2}\right)^{\frac{1}{\hat{b}+1}}$
Mxi $y = a + \frac{b}{x}$	$\frac{-b}{x^2}$	$\frac{2b}{x^3}$	$\sqrt{\hat{b}}$	$\sqrt{\frac{\hat{b}(x_2 - x_1)}{(y_1 - y_2)}}$
MMi $y = \frac{a+x}{bx}$	$\frac{-a}{bx^2}$	$\frac{2a}{bx^3}$	$\sqrt{\frac{\hat{a}}{\hat{b}}}$	$\sqrt{\frac{\hat{a}(x_2 - x_1)}{\hat{b}(y_1 - y_2)}}$
Mhi $y = \frac{1}{a+bx}$	$\frac{-b}{(a+bx)^2}$	$\frac{2b}{(a+bx)^3}$	$\frac{\sqrt{\hat{b}} - \hat{a}}{\hat{b}}$	$\frac{1}{\hat{b}}\left[\sqrt{\frac{\hat{b}(x_2 - x_1)}{(y_1 - y_2)}} - \hat{a}\right]$
Mrx $y = a + \frac{b}{\sqrt{x}}$	$\frac{-b}{2\sqrt{x^3}}$	$\frac{3b}{4\sqrt{x^5}}$	$\left(\frac{\hat{b}^2}{5}\right)^{\frac{1}{3}}$	$\left(\frac{\hat{b}(x_2 - x_1}{2(y_1 - y_2}\right)^{\frac{2}{3}}$
Mex $y = \frac{a}{\exp(bx)}$	$\frac{-ab}{\exp(bx)}$	$\frac{ab^2}{\exp(bx)}$	$\frac{\ln(\sqrt{2}\,\hat{a}\hat{b})}{\hat{b}}$	$\frac{1}{\hat{b}}\ln\left(\frac{\hat{a}\hat{b}(x_2 - x_1}{(y_1 - y_2}\right)$
Me2 $y = \frac{1+\exp(-ax)}{b}$	$\frac{-a\exp(-ax)}{b}$	$\frac{a^2\exp(-ax)}{b}$	$\frac{-1}{\hat{a}}\ln\left(\frac{\hat{b}}{\sqrt{2}\,\hat{a}}\right)$	$\frac{-1}{\hat{a}}\ln\left(\frac{\hat{b}(y_1 - y_2)}{\hat{a}(x_2 - x_1)}\right)$
Mei $y = a + \frac{b}{\exp(x)}$	$\frac{-b}{\exp(x)}$	$\frac{b}{\exp(x)}$	$\ln\left(\sqrt{2}\,\hat{b}\right)$	$\ln\left(\frac{\hat{b}(x_2 - x_{1)}}{(y_1 - y_2)}\right)$

$\hat{a}$ e $\hat{b}$ represent the estimates of parameters *a* and *b*, respectively.

3.2 Applications

For the coffee assay, analyzing variables MSPA (Table 3) and MST (Table 4), optimum plot sizes ranged from 4.6 BU to 14.9 BU, according to the models used. It was observed that models Mxi, MMi, Me2 and Mei estimated plot sizes equal to or smaller than the estimates obtained by MMC. As some authors (LEITE *et al*., 2006; PARANAÍBA *et al*., 2009) emphasize that MMC underestimates optimum plot size; these models, in this aspect, lose in quality, because they estimate even smaller plots. For MST (Table 4), MMC estimated PS equal to 6.4 BU, whereas, for Mxi, MMi and Me2 models, the estimation of PS was 5.7 BU and Mei model estimated PS of 4.7 BU, lower than that obtained by MMC. On the other hand, the other proposed models estimated plot sizes (PS) ranging from 8.4 BU to 14.9 BU.

These results are more consistent with commonly used plot sizes, such as by Firmino *et al* (2012) who used nine plants; Santinato *et al*. (2014) and Dias *et al.* (2015), plots formed by eight coffee seedlings, and by Pereira et al (2017) with seven plants. The results of plot sizes (PS) obtained by the different methods also do not differ from those found in other crops or experiments, such as Henriques Neto *et al.* (2009), in the estimation of PS for irrigated wheat production, Paranaíba *et al*. (2009), in the proposition of the CV model and Brito *et al.* (2012), in the estimation of PS for plant height in papaya. The use of different methods in general provides different results.

Information on plot size with coffee seedlings is also variable, as was the case with estimates obtained by the models; it is clear that the ideal would be for the results of the methods to converge to the same value. PS estimates obtained for MST (Table 4) ranged from 5.7 BU to 14.4 BU, according to the model and method used. These estimates are consistent with work by Gonçalves *et al.* (2009), who used plots of 13 seedlings, of which five were used for evaluations; Meneghelli *et al.* (2016), using plots of 10 seedlings; Aparecido *et al.* (2017), who used plot consisting of three rows with nine plants in each, adopting the five central plants as useful area, and lower than those used by Berilli *et al.* (2014) and Oliveira and Miglioranza (2015), who used 30 seedlings, and by Alves *et al*. (2016) and Meneghelli *et al*. (2017) who used plots with 20 seedlings.

These results are similar to those found by Vieira and Silva (2008) and Henriques Neto *et al.* (2009), who indicated that for each situation, crop management, there must be a proper PS, which should not be generalized. This was the case with the various estimation methods. If one wants to have a more reliable experiment from the point of view of practical needs and evaluations, models that estimated higher PS should be selected. In this sense, for the dry mass data (Tables 3 and 4), Mex model provided the highest PS, followed by the mixed algebraic method, which should only be used in a situation of greater abundance of resources.

Table 3 - Estimates of parameters of models used and respective asymptotic standard errors, residual standard deviation (RSD), Akaike information criterion (AIC), adjusted coefficient of determination (R_a^2) and plot size in number of basic units, using the proposed methods (x_c) and algebraic method (x_a) applied to each model, for dry mass of shoots (MSPA) of coffee seedlings.

Model	Parameters		Quality criterion			Plot Size (BU)	
	a	b	RSD	AIC	R_a^2	x_c	x_a
MMC $y = \frac{a}{x^b}$	34.5422	0.4663	1.04	44.6	0.98	6.1	12.4
	(0.8641)	(0.0169)					
Mxi $y = a + \frac{b}{x}$	7.8960	29.9900	1.87	61.2	0.94	5.5	8.6
	(0.6320)	(1.9330)					
MMi $y = \frac{a+x}{b\,x}$	3.7979	0.1266	1.87	61.2	0.94	5.5	8.6
	(0.4931)	(0.0101)					
Mhi $y = \frac{1}{a+b\,x}$	0.02512	0.00626	2.69	71.3	0.89	8.6	15.9
	(0.0027)	(0.00091)					
Mrx $y = a + \frac{b}{\sqrt{x}}$	1.3309	33.2208	0.91	40.8	0.99	6.0	11.9
	(0.4536)	(1.0158)					
Mex $y = \frac{a}{\exp(b\,x)}$	29.4018	0.08213	4.58	86.2	0.67	14.9	21.8
	(3.4532)	(0.0197)					
Me2 $y = \frac{1+\exp(-a\,x)}{b}$	0.18136	0.08454	5.36	90.6	0.55	6.1	9.2
	(0.1894)	(0.0161)					
Mei $y = a + \frac{b}{\exp(x)}$	10.8703	73.2878	3.80	80.9	0.77	4.6	6.1
	(1.1020)	(10.4280)					

Table 4 - Estimates of parameters of models used and respective asymptotic standard errors, residual standard deviation (RSD), Akaike information criterion (AIC), adjusted coefficient of determination (R_a^2) and plot size in number of basic units by the proposed methods (x_c) and algebraic method (x_a) applied to each model, for total dry matter (TDM) values of coffee seedlings.

Model	Parameters		Quality criterion			Plot Size (BU)	
	a	b	RSD	AIC	R_a^2	x_c	x_a
MMC $y = \frac{a}{x^b}$	35.7123	0.5394	0.86	39.38	0.99	6.4	11.6
	(0.7391)	(0.0155)					
Mxi $y = a + \frac{b}{x}$	6.2669	32.4587	2.01	63.16	0.94	5.7	8.6
	(0.6789)	(2.0764)					
MMi $y = \frac{a+x}{bx}$	5.17942	0.15957	2.01	63.16	0.94	5.7	8.6
	(0.8071)	(0.0173)					
Mhi $y = \frac{1}{a+bx}$	0.02157	0.00819	2.07	63.99	0.94	8.4	14.1
	(0.0022)	(0.00089)					
Mrx $y = a + \frac{b}{\sqrt{x}}$	-0.8381	35.9533	0.96	42.27	0.99	6.4	11.9
	(0.4781)	(1.0706)					
Mex $y = \frac{a}{\exp(bx)}$	32.3342	0.1147	4.06	82.77	0.78	14.4	18.6
	(3.5706)	(0.0232)					
Me2 $y = \frac{1+\exp(-ax)}{b}$	0.17528	0.09083	6.29	95.09	0.47	5.7	8.5
	(0.2308)	(0.0218)					
Mei $y = a + \frac{b}{\exp(x)}$	9.4894	79.2287	4.13	83.26	0.77	4.7	5.3
	(1.1970)	(11.3200)					

For the radish trial, the Mrx model stood out as having the best adjustment quality, followed by the MMC model. For this assay, plot sizes ranged from 4.6 BU to 20.5 BU (Tables 5 and 6), with an estimate of 7.7 BU for the tuber mass by the Mrx model and 6.0 BU for the diameter, which are values close to those obtained by the MMC method. For this trial, there was greater discrepancy between values estimated for the PS in relation to variables mass and diameter.

Table 5 - Estimates of parameters of models, respective asymptotic standard errors, residual standard deviation (RSD), Akaike information criterion (AIC), adjusted coefficient of determination (R_a^2) and plot size in number of basic units by methods proposed (x_c) and algebraic method (x_a) applied to each model, for radish mass values

Model	Parameters		Quality criterion			Plot Size (BU)	
	a	b	RSD	AIC	R_a^2	x_c	x_a
MMC $y = \frac{a}{x^b}$	49.6486	0.4326	0.88	39.96	0.99	7.8	24.0
	(0.7311)	(0.0091)					
Mxi $y = a + \frac{b}{x}$	10.8589	44.1348	3.48	78.48	0.92	6.6	14.0
	(1.1090)	(3.5210)					
MMi $y = \frac{a+x}{bx}$	4.0643	0.0921	3.48	78.48	0.92	6.6	14.0
	(0.6510)	(0.0094)					
Mhi $y = \frac{1}{a+bx}$	0.0187	0.0034	4.10	83.09	0.88	11.6	30.5
	(0.0019)	(0.0005)					
Mrx $y = a + \frac{b}{\sqrt{x}}$	2.8557	47.5936	0.54	26.60	0.99	7.7	22.3
	(0.2378)	(0.5743)					
Mex $y = \frac{a}{\exp(bx)}$	40.4990	0.0603	6.76	97.08	0.69	20.5	39.5
	(4.8323)	(0.0162)					
Me2 $y = \frac{1+\exp(-ax)}{b}$	0.1236	0.0632	8.39	103.15	0.52	8.2	17.5
	(0.1428)	(0.0121)					
Mei $y = a + \frac{b}{\exp(x)}$	14.3454	108.0365	6.34	95.26	0.73	5.0	6.2
	(1.8150)	(17.3010)					

Plot sizes in the radish crop are also variable according to the type of experiment. Reis *et al.* (2012) used plots of 80 plants, of which 36 central plants constituted the useful area; using 1m^2 plots. Dutra *et al.* (2014) considered the central region of 0.4 × 0.4 m containing 15 plants as the useful area for the evaluations; and Castro *et al*. (2016) used an experimental unit of dimensions 1 × 0.8 m, considering 10 central plants as the useful area per plot. For the greenhouse experiment, Caetano *et al*. (2015) used plot consisting of one pot with capacity of 8 L of soil, with five plants in each pot.

Table 6 - Estimates of parameters of models, respective asymptotic standard errors, residual standard deviation (RSD), Akaike information criterion (AIC), adjusted coefficient of determination (R_a^2) and plot size in number of basic units by methods proposed (x_c) and algebraic method (x_a) applied to each model, for radish diameter values

Model	Parameters		Quality criterion			Plot Size (BU)	
	a	b	RSD	AIC	R_a^2	x_c	x_a
MMC $y=\frac{a}{x^b}$	32.6071 (0.6692)	0.4619 (0.0134)	0.79	37.10	0.99	5.9	22.7
Mxi $y=a+\frac{b}{x}$	6.2303 (0.9812)	29.5984 (3.1167)	3.08	75.06	0.86	5.4	13.7
MMi $y=\frac{a+x}{bx}$	4.7507 (1.1034)	0.1605 (0.0252)	3.08	75.06	0.86	5.4	13.7
Mhi $y=\frac{1}{a+bx}$	0.02829 (0.0022)	0.00566 (0.00069)	1.95	62.33	0.94	8.3	28.5
Mrx $y=a+\frac{b}{\sqrt{x}}$	0.64002 (0.4053)	32.6001 (0.9789)	0.93	41.53	0.99	6.0	22.1
Mex $y=\frac{a}{\exp(bx)}$	26.7430 (2.6966)	0.0651 (0.0145)	3.66	79.89	0.81	13.8	36.9
Me2 $y=\frac{1+\exp(-ax)}{b}$	0.10563 (0.1435)	0.1036 (0.0233)	6.07	94.06	0.47	3.5	17.7
Mei $y=a+\frac{b}{\exp(x)}$	8.6147 (1.4050)	71.2301 (13.393)	4.90	88.09	0.65	4.6	6.1

According to the adjustment quality evaluation criteria (Tables 3, 4, 5 and 6), the models that presented the best quality were MMC and Mrx, with lower RSD and AIC estimates and higher determination coefficients. On the other hand, Mex model did not present good adjustment quality, with higher of RSD and AIC estimates, and was also the model that estimated the largest plot size in number of basic units. Thus, Mrx stands out for having lower estimate of these criteria, and also for presenting PS estimates close to those obtained by the MMC method, and with an intermediate value to those obtained by the other models. This model has the advantage that the estimation of PS depends on only one parameter.

For the examples given, it is difficult to highlight only one of the models. In both uniformity tests, the Mrx model was highlighted, with lower RSD and intermediate PS. The Mxi, MMi and Mei models presented lower PS estimates and intermediate RSD values; in some situations of shortage of material, they could be used.

The calculation of the plot size by the modified maximum curvature technique has the advantage of establishing a relationship between coefficient of variation (measure of experimental precision) and plot size by means of regression equation, whose adjustment is generally very good, with high determination coefficient values, as most of those found in this work, increasing the reliability of estimates. Thus, based on plot size estimates, ranging from 5.5 to 15.0 BU, obtained by the proposed models, it is suggested to use an intermediate value as the appropriate plot size.

The exponential model Mei presented more consistent plot size estimates in relation to the different variables and between MC technique and algebraic method. However, it presented the worst model quality in terms of fitted indexes.

In the algebraic analysis, quite different values were obtained, which can be attributed to the characteristics inherent of each model, because among variables in each trial, estimates did not differ significantly. Using this technique, for models like Mex and Mhi, an overestimation of PS occurred, whereas for Mxi and MMi models, PS estimations are more coherent.

In addition, and perhaps the most important point, is that this technique depends on the amplitude of x_1 and x_2 and y_1 and y_2 values; the farther x_2 is from x_1, higher estimates are expected for PS. The plot size estimators (x_a) shown in Table 2 are dependent on the magnitude of values observed in the trials. But, this technique can be quite useful according to the research needs in using larger plot sizes.

The results obtained by the methods and models presented reflect the effects of the variability of variables obtained in the uniformity trials. Thus, one should select methodologies that consider the coherence of the objectives of each research, combined with the researcher's common sense and the tests and statistical criteria that are intended to be used.

Conclusions

The models presented are useful to calculate the optimum plot size, generally estimating plots larger than those obtained by the modified model of maximum curvature (MMC).

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MORAIS, A.R.; VILLA, F.; GONZALEZ, G.G.H.; MORAIS, E.C. Modelos para a estimação do tamanho de parcelas em experimentos. *Revista Brasileira de Biometria*, Lavras, v.36, n.2, p.258-275, 2018.

▪ *RESUMO: A utilização de modelos estatísticos na estimação de tamanho de parcela experimental é uma prática que contribui para o planejamento experimental por meio da escolha de um tamanho que propicie melhor eficiência na comparação dos tratamentos. Este trabalho teve por objetivo apresentar alguns modelos estatísticos, com a finalidade de utilizá-los como alternativas na determinação do tamanho ótimo de parcelas em experimentos. Foram propostos alguns modelos não lineares com configuração simples e semelhante à do modelo proposto pela técnica da máxima curvatura modificada, os quais são deriváveis e possuem função de curvatura. Foi* *obtida a função de curvatura para cada modelo e por meio do ponto crítico da derivada da função de curvatura obtiveram-se os estimadores do tamanho da parcela. Os modelos se mostraram viáveis para serem utilizados na estimação de tamanho de parcelas com estimadores mais simples do que aquele obtido por Meier e Lessman. Como ilustração, para aplicação da proposta e comparação com o método da máxima curvatura modificado foram utilizados dados de dois ensaios de uniformidade. As estimativas do tamanho ótimo de parcela variaram de acordo com o modelo e método.*

- *PALAVRAS-CHAVE: Técnica da máxima curvatura; precisão experimental; erro experimental; coeficiente de variação; ensaio de uniformidade.*

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