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ARTICLE

Performance of the Principal Components Estimation Method on Factor Analysis Quality without and with Varimax Rotation

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Abstract

The objective of this work was to evaluate the performance of the principal components estimation method on the quality of estimates of some parameters of exploratory factor analysis (EFA), without and with varimax rotation. For this purpose, 18 parametric correlation matrices (ρ) were imposed. These matrices were obtained from combinations of six different values of parametric commonalities of four normally distributed random variables with three different proportions of distances between the parametric factor loadings of the first two orthogonal factors. For each matrix ρ, the following parameters were defined: the first two eigenvalues (λ_1 and λ_2), the matrix of factor loadings (Γ), the four commonalities (h_1^2, h_2^2, h_3^2 and h_4^2) the matrix of the specific factors (Ψ). After the 36 factor analyses, the respective estimates of these parameters were obtained, and their respective absolute deviations between the estimates obtained a posteriori and the parameters are known a priori were evaluated. With the results of the Student's t-test at 5% significance applied to the response surface analysis, it could have been inferred that the principal components estimation method for estimating orthogonal factors was not adequate and the varimax rotation improved relatively little to the quality of the AFE estimates.

Keywords: Commonality; Correlation; Multivariate analysis.

1. Introduction

Exploratory factor analysis (EFA) is a multivariate statistical method, whose objective is to determine the number and core of latent variables or factors that best represent the variances and covariances among the observed variables (Brown, 2015).

Otherwise, it can be stated that the EFA is a statistical method of interdependence that reproduces the relationships between the variables observed in a smaller group of factors, where each observed variable defined as their linear combination, in such a way that this combination represents the decomposition of variances and covariances between these variables. This means that each factor represents a set of observed variables that are highly correlated with each other, so that they incorporate the latent dimensions of the variables in their group, returning a parsimonious model. Therefore, the objective of EFA is to simplify the dataset in order to the interpretation of the original variables and the relationships between them clearer and more accurate.

EFA is one of the most used multivariate statistical methods for data analysis in several areas of knowledge, such as agronomy, zootechnics, ecology, forestry, medicine, and social sciences (Hongyu, 2018).

In Agricultural Sciences, some works have been carried out to determine the combining ability of l broiler breeder lines (Abreu *et al.*, 1999), to assist in the genomic selection of pigs (Teixeira *et al.*, 2015), to assess the spatial dependence of chemical attributes of the soil (Almeida & Guimarães, 2016) and to characterize the factors inherent to the occurrence of dwarf pequi (Almeida, 2021). In this field of knowledge, EFA is widely applied among breeders for the use of an orthogonal factorial model, that is, of uncorrelated factors that enable a solution to the problem of multicollinearity (Ferreira *et al.*, 2005).

The solution of an EFA depends on the covariance matrix and, therefore, is not invariant to scale changes. Furthermore, it is not unique, that is, there are different solutions capable of reproducing the same matrix, and this will depend on the number of factors in the model, the estimation method, and the rotation used in the analysis. According to Ferreira (2018), the non-uniqueness of the parameters of the factorial model can be used favorably to improve the interpretation of the factors. On the other hand, it can generate doubts because it allows different results for the same data set.

It should be observed that EFA is often confused with principal component analysis (PCA). This fact occurs because one of the methods of extracting factors is the method of principal components. In addition, PCA also aims to reduce data, but EFA is not limited to this purpose only (Hauck Filho & Valentini, 2020; Rogers, 2022).

The mathematical complexity and diversity of options for each EFA procedure make understanding this technique difficult. On the other hand, with the advancement of technology and software, the popularity of using EFA has increased, as it has become more accessible.

According to Costello and Osborne (2005), in a survey of over 1,700 works that used some form of EFA, well over half reported the use of principal components and varimax rotation for data analysis. Moreover, in a large part of the published works in which the EFA is performed, the estimation method used is not reported, but only the performance or not of the rotation and the rotation method applied. This suggests, therefore, that in these works, the principal components estimation method was used, given that for EFA, it is the simplest (Ferreira, 2018) and is usually the standard configuration of the software that contemplates this analysis (Costello; Osborne, 2005).

In practice, the varimax criterion is the most used and usually produces factors that are not correlated with each other and simpler solutions than the other methods (Mingoti, 2013).

Consequently, the objective of this work was to evaluate the performance of the principal components method used in EFA, with and without varimax rotation, in providing adequate estimates of the parameters of the orthogonal factorial model according to different configurations of relationships between four normally distributed random variables.

1.1 Exploratory Factor Analysis

To start an EFA, the interdependence structure of the data set must be verified. It can be represented by the covariance matrix between the original variables $Y_1, Y_2, \ldots, Y_p$ ($\mathbf{\Sigma_Y}$) or standardized $Z_1, Z_2, \ldots, Z_p$ ($\mathbf{\Sigma_Z}$). It is noteworthy that when the standardization of the original variables is based on their respective means and standard deviations, new variables centered on zero and with variances equal to one are obtained. In this case, the matrix of covariances between the standardized variables ($\mathbf{\Sigma_Z}$) coincides with the matrix of correlations between the original variables ($\boldsymbol{\rho}_\mathbf{Y}$).

Usually, it is decided to analyze the variables on a standardized scale, that is, to work with the correlation matrix between the original variables ($\mathbf{\Sigma_Z} = \boldsymbol{\rho}_\mathbf{Y}$) to minimize the effect of the difference between the measurement units or of the scales of the original variables.

Thus, the factorial model with k ($k \leq p$) factors is defined, as follows:

$$
\begin{aligned}
&Z_1 = \gamma_{11}F_1 + \gamma_{12}F_2 + \ldots + \gamma_{1k}F_k + \varepsilon_1;\\
&Z_2 = \gamma_{21}F_1 + \gamma_{22}F_2 + \ldots + \gamma_{2k}F_k + \varepsilon_2;\\
&\ldots\\
&Z_p = \gamma_{p1}F_1 + \gamma_{p2}F_2 + \ldots + \gamma_{pk}F_k + \varepsilon_p.
\end{aligned}
$$

In matrix terms, this definition can be expressed by:

$$\mathbf{z} = \mathbf{\Gamma F} + \boldsymbol{\varepsilon} \Rightarrow \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_p \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1k} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{p1} & \gamma_{p2} & \cdots & \gamma_{pk} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_p \end{bmatrix}, \text{ where:}$$

$\mathbf{\Gamma}$ = matrix (p × k) of the factor loadings referring to the correlations between the original variables and the factors;

$\mathbf{F}$ = vector (k x 1) of common factors (factors); and

$\boldsymbol{\varepsilon}$ = vector (p x 1) of random errors (specific factors).

Thus, it results in:

$\mathrm{Var}(Z_w) = \gamma_{w1}^2 + \gamma_{w2}^2 + \ldots + \gamma_{wk}^2 + \psi_w = h_w^2 + \psi_w = 1$, where:

$Z_w = \frac{Y_w - \mu_w}{\sigma_w}$;

μ_w = average of the original variable Y_w;

σ_w =standard deviation of the original variable Y_w;

γ_{wj} =factor loading or correlation between the original variable Y_w and the factor F_j;

$h_w^2 = \sum_{j=1}^{k} \gamma_{wj}^2$= commonality or proportion of the variance of the standardized variable Z_w explained by the k common factors (factors); and

$\psi_w = 1 - h^2{}_w$ = specificity or variance explained by the specific factor ε_w (w = 1, 2, ..., p and j = 1, 2, ..., k).

To proceed with the estimation of the factorial model, some assumptions are necessary:

$E(\mathbf{F}) = E(\boldsymbol{\varepsilon}) = \mathbf{0}$;

$\mathrm{Cov}(\mathbf{F}) = E(\mathbf{FF'}) = \mathbf{I}$;

$$\mathrm{Cov}(\boldsymbol{\varepsilon}) = E(\boldsymbol{\varepsilon\varepsilon'}) = \mathbf{\Psi} = \begin{bmatrix} \psi_1 & 0 & \ldots & 0 \\ 0 & \psi_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \psi_p \end{bmatrix}; \text{ and}$$

$\mathrm{Cov}(\mathbf{F},\boldsymbol{\varepsilon}) = E(\boldsymbol{\varepsilon}\mathbf{F'}) = \mathbf{0}$.

If such properties are observed in the model, it is denominated an orthogonal factorial model and, in this case, the matrix $\mathbf{\Sigma_Z} = \boldsymbol{\rho}_\mathbf{Y}$ ($\boldsymbol{\rho}$) can be decomposed, as follows:

$\boldsymbol{\rho} = \mathbf{\Gamma\Gamma'} + \mathbf{\Psi}$.

The principal components estimation method is based on the spectral decomposition of the matrix $\boldsymbol{\rho}$, which guarantees that every symmetric matrix can be decomposed as follows:

$\boldsymbol{\rho} = \mathbf{P\Lambda P'} = \underbrace{\mathbf{P\Lambda^{1/2}}}_{\mathbf{\Gamma}}\underbrace{\mathbf{\Lambda^{1/2}P'}}_{\mathbf{\Gamma'}} = \mathbf{\Gamma\Gamma'}$, where:

$\mathbf{\Lambda}$ = matrix (p x p) diagonal with the eigenvalues of the matrix (p × p) $\boldsymbol{\rho}$;

$\mathbf{P}$ = matrix (p x p) with the normalized eigenvectors of the matrix (p × p) $\boldsymbol{\rho}$, in its columns, associated with each eigenvalue; and

$\mathbf{\Gamma} = \mathbf{P\Lambda^{1/2}}$ = matrix (p x k) of factor loadings.

Therefore, the matrices $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ are estimated as follows:

$\mathbf{R} = \hat{\mathbf{\Gamma}}\hat{\mathbf{\Gamma}}' + \hat{\mathbf{\Psi}}$, where:

$\mathbf{R}$ = matrix (p x p) of sample covariances between standardized variables or sample correlations between original variables;

$$\widehat{\Gamma} = \begin{bmatrix} \sqrt{\hat{\lambda}_1}\hat{e}_{11} & \sqrt{\hat{\lambda}_2}\hat{e}_{12} & \cdots & \sqrt{\hat{\lambda}_k}\hat{e}_{1k} \\ \sqrt{\hat{\lambda}_1}\hat{e}_{21} & \sqrt{\hat{\lambda}_2}\hat{e}_{22} & \cdots & \sqrt{\hat{\lambda}_k}\hat{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\hat{\lambda}_1}\hat{e}_{p1} & \sqrt{\hat{\lambda}_2}\hat{e}_{p2} & \cdots & \sqrt{\hat{\lambda}_k}\hat{e}_{pk} \end{bmatrix};$$

$\widehat{\boldsymbol{\Psi}} = \text{diag}\left[\mathbf{R} - \widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}'\right]$ = matrix (p x p) of sample variances of specific factors;

$\hat{h}_w^2 = \sum_{j=1}^{k} \hat{\gamma}_{wj}^2$ = estimate of the commonality h_w^2 of the standardized variable Z_w;

$\hat{\lambda}_j = \sum_{w=1}^{p} \hat{\gamma}_{wj}^2$ = estimate of the eigenvalue λ_j;

$\hat{\gamma}_{jw} = \sqrt{\hat{\lambda}_j}\hat{e}_{wj}$ = estimate of the factor loading γ_{wj}; and

$\hat{e}_{wj}$ = estimate of the e_{wj} coefficient of the normalized eigenvector $(p \times 1)$ $\mathbf{e_j}$ (w = 1, 2, ..., p and j = 1, 2, ..., k).

So $\widehat{\boldsymbol{\Psi}} = \text{diag}\left[\mathbf{R} - \widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}'\right]$, an approximation of the matrix $\boldsymbol{\rho}$ is obtained, such that: $\mathbf{R} \cong \widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}' + \widehat{\boldsymbol{\Psi}}$.

This occurs because the random variation not common to the factors and the contribution of the last p – k normalized eigenvectors and eigenvectors of **R** are disregarded.

Thus, one of the methods to assess the goodness of fit of the factorial model estimated by the principal components method consists of calculating the residual sample matrix given by: $\mathbf{MRes} = \mathbf{R} - (\widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}' + \widehat{\boldsymbol{\Psi}})$.

The closer the values of the **MRes** matrix are to zero, the more the $\widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}' + \widehat{\boldsymbol{\Psi}}$ approached the **R** matrix and, consequently, the greater its representativeness (MINGOTI, 2013).

It is also known that the total variance can be calculated by tracing the matrix **R**. Therefore, the estimate of the proportion of the total variation explained by the F_j factor is defined as:

$\frac{\sum_{w=1}^{p} \hat{\gamma}_{wj}^2}{\text{tr}(\mathbf{R})} = \frac{\sum_{w=1}^{p} \hat{\gamma}_{wj}^2}{p}$, towards j = 1, 2, ..., k.

The principal components estimation method received this name because it is based on the use of eigenvalues and normalized eigenvectors of the R matrix to estimate the factor loadings.

Thus, the estimate of the factor model with the factors $F_1, F_2, ..., F_k$ ($k \leq p$) is defined, as:

$$Z_1 = \hat{\gamma}_{11}F_1 + \hat{\gamma}_{12}F_2 + \ldots + \hat{\gamma}_{1k}F_k;$$
$$Z_2 = \hat{\gamma}_{21}F_1 + \hat{\gamma}_{22}F_2 + \ldots + \hat{\gamma}_{2k}F_k;$$
$$\ldots$$
$$Z_p = \hat{\gamma}_{p1}F_1 + \hat{\gamma}_{p2}F_2 + \ldots + \hat{\gamma}_{pk}F_k.$$

To facilitate the interpretation of the factors and seek a simpler structure for the matrix $\boldsymbol{\Gamma}$ of the factor loadings, it is possible to perform an orthogonal rotation of the factors, to preserve the original orientation between them. The objective of the factorial rotation is to simplify the lines and columns of this matrix, to make a maximum of only one factor in each line, and to try to reduce or approximate, as much as possible, the other factors to the zero values, as many times, the analyzed variables present high factor loadings in more than one factor (Damásio, 2012).

A square matrix **T** is said to be an orthogonal matrix if and only if $\mathbf{TT}' = \mathbf{T}'\mathbf{T} = \mathbf{I}$. Thus, the orthogonal transformation results from the multiplication of any orthogonal matrix **T** by the matrix $\boldsymbol{\Gamma}$ of parametric factor loadings, as follows:

$\boldsymbol{\Gamma}^* = \boldsymbol{\Gamma}\mathbf{T}$, where:

$\boldsymbol{\Gamma}^*$ = matrix (p x k) of rotated and parametric factor loadings.

The varimax criterion provides estimates of the rotated factor loadings that maximize the following expression:

$v^* = \frac{1}{p^2}\sum_{j=1}^{k}\left[\sum_{w=1}^{p}\hat{\gamma}_{wj}^4 - \frac{1}{p}\left(\sum_{w=1}^{p}\hat{\gamma}_{wj}^2\right)^2\right]$, where:

$\hat{\gamma}^*_{wj} = \frac{\hat{\gamma}_{jw}}{\hat{h}_w}$ = estimate of the rotated factor loading or the correlation between the original variable Y_w and the rotated factor F^*_j; and

$\hat{h}_w$ = estimate of the square root of the commonality or the proportion of the standard deviation of the standardized variable Z_w explained by the k common factors (factors) (w = 1, 2, ..., p and j = 1, 2, ..., k).

According to Johnson and Wichern (2002), all factor loadings obtained from an orthogonal transformation of the initial factor loadings have the same ability to reproduce the matrix $\boldsymbol{\rho}$. Mingoti (2013) says that orthogonal rotation does not change the fit of the factorial model determined from the $\boldsymbol{\Gamma}$ e $\boldsymbol{\Psi}$, matrices, as the **MRes** matrix, the estimates of commonalities and specific variances remain unchanged. Consequently, only the estimates of the eigenvalues, factor loadings, and normalized eigenvectors will change with rotation. Likewise, the matrix $\boldsymbol{\Gamma}$ is estimated as follows:

$\mathbf{R} \cong \hat{\boldsymbol{\Gamma}}^* \hat{\boldsymbol{\Gamma}}^{*\prime} + \hat{\boldsymbol{\Psi}}$, where:

$$\hat{\boldsymbol{\Gamma}}^* = \begin{bmatrix} \sqrt{\hat{\lambda}^*_1}\hat{e}^*_{11} & \sqrt{\hat{\lambda}^*_2}\hat{e}^*_{12} & \cdots & \sqrt{\hat{\lambda}^*_k}\hat{e}^*_{1k} \\ \sqrt{\hat{\lambda}^*_1}\hat{e}^*_{21} & \sqrt{\hat{\lambda}^*_2}\hat{e}^*_{22} & \cdots & \sqrt{\hat{\lambda}^*_k}\hat{e}^*_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\hat{\lambda}^*_1}\hat{e}^*_{11} & \sqrt{\hat{\lambda}^*_2}\hat{e}^*_{p2} & \cdots & \sqrt{\hat{\lambda}^*_k}\hat{e}^*_{pk} \end{bmatrix};$$

$\hat{\lambda}^*_j = \sum_{w=1}^{p} \hat{\gamma}^{*2}_{wj}$ = estimate of the rotated eigenvalue λ^*_j;

$\hat{\gamma}^*_{wj} = \sqrt{\hat{\lambda}^*_j}\hat{e}^*_{wj}$ = estimate of the rotated factor loading γ^*_{wj}; e

$\hat{e}^*_{wj}$ = estimate of the rotated coefficient e^*_{wj} of the rotated normalized eigenvector (p × 1) $\mathbf{e}^*_j$ (w = 1, 2, ..., p and j = 1, 2, ..., k).

Thus, the estimate of the factorial model with the rotated factors $F^*_1, F^*_2, ..., F^*_k$ (k ≤ p) defined as follows:

$Z_1 = \hat{\gamma}^*_{11}F^*_1 + \hat{\gamma}^*_{12}F^*_2 + \ ... \ + \hat{\gamma}^*_{1k}F^*_k$;

$Z_2 = \hat{\gamma}^*_{21}F^*_1 + \hat{\gamma}^*_{22}F^*_2 + \ ... \ + \hat{\gamma}^*_{2k}F^*_k$;

...

$Z_p = \hat{\gamma}^*_{p1}F^*_1 + \hat{\gamma}^*_{p2}F^*_2 + \ ... \ + \hat{\gamma}^*_{pk}F^*_k$.

2. Materials and Methods

2.1 Factor model

Based on the theory of factor analysis, it was decided in this work to determine criteria to theoretically establish some EFA parameters, namely: matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$, which allowed the calculation of different correlation matrices and with *a priori* specific characteristics, to evaluate *a posteriori*, the results obtained by the EFA.

Thus, it was stipulated four original and random variables with a distribution of multivariate normal probabilities, such as:

Y_1, Y_2, Y_3 e Y_4 ~ N_4 ($\boldsymbol{\mu}_\mathbf{Y}$; $\boldsymbol{\Sigma}_\mathbf{Y}$), where:

$\boldsymbol{\mu}_\mathbf{Y}$ = vector (4 × 1) of parametric means of the four original variables; and

$\boldsymbol{\Sigma}_\mathbf{Y}$ = matrix (4 × 4) of parametric covariances between the four original variables.

To avoid scale problems, the study of the original variables Y_1, Y_2, Y_3, and Y_4, was carried out based on the respective standardized variables Z_1, Z_2, Z_3, and Z_4 where:

$Z_w = \frac{Y_w - \mu_w}{\sigma_w}$, towards w = 1, 2, 3 e 4.

Once defined, the four standardized and random variables followed a multivariate normal probability distribution:

Z_1, Z_2, Z_3 e Z_4 ~ N_4 ($\boldsymbol{\mu}_\mathbf{Z}$; $\boldsymbol{\Sigma}_\mathbf{Z}$), where:

$\boldsymbol{\mu}_\mathbf{Z} = \mathbf{0}$ = vector (4 × 1) of parametric means equal to zero of the four standardized variables; and

$\mathbf{\Sigma_Z} = \boldsymbol{\rho}_\mathbf{Y}$ = matrix (4 × 4) of parametric covariances between the four standardized variables or of the parametric correlations between the four original variables.

It is important to emphasize that for the factorial model to be useful in practice, a number of factors smaller than the number of original variables must be considered. Thus, in order to obtain a parsimonious factorial model, two factors (F1 and F2) were constructed, thus defining the orthogonal factorial model based on the four standardized variables, as follows:

$Z_1 = \gamma_{11}F_1 + \gamma_{12}F_2 + \varepsilon_1$;

$Z_2 = \gamma_{21}F_1 + \gamma_{22}F_2 + \varepsilon_2$;

$Z_3 = \gamma_{31}F_1 + \gamma_{32}F_2 + \varepsilon_3$; and

$Z_4 = \gamma_{41}F_1 + \gamma_{42}F_2 + \varepsilon_4$, where:

γ_{wj} = parametric factor loading or parametric correlation between the original variable Y_w and the common factor (factor) F_j; and

ε_w = specific parametric factor associated with the standardized variable Z_w (w = 1, 2, 3 and 4 and j = 1 and 2).

In matrix form, it is observed:

$$\mathbf{z} = \mathbf{\Gamma F} + \boldsymbol{\varepsilon} \Rightarrow \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \\ \gamma_{41} & \gamma_{42} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}, \text{ where:}$$

$\mathbf{\Gamma}$ = matrix (4 × 2) of parametric factor loadings.

2.2 Factors

To obtain the $\mathbf{\Gamma}$ matrices, defined by the first two orthogonal factors (F_1 e F_2), six different parametric commonalities (0.49; 0.56; 0.64; 0.72; 0.81 and 0.90) associated with the four standardized variables (Z_1, Z_2, Z_3, and Z_4) were equally ($h_1^2 = h_2^2 = h_3^2 = h_4^2$) established.

$h_w^2 = \gamma_{w1}^2 + \gamma_{w2}^2$, for w = 1, 2, 3 and 4, where:

h^2_w = parametric commonality of the standardized variable Z_w;

γ_{w1} = parametric factor loading or parametric correlation between the original variable Y_w and the factor F_1; and

γ_{w2} = parametric factor loading or parametric correlation between the original variable Y_w and the F_2 factor.

In addition, for the selection of the eight parametric factor loadings or the eight parametric correlations between the original variable Y_w (w = 1, 2, 3 and 4) and the factors F_1 (γ_{w1}) and F_2 (γ_{w2}), three proportions of the distances between the parametric factor loadings of the two factors (0.20; 0.45; and 0.81), for each of the six parametric commonalities h_w^2, respectively, were established. These three distances were chosen to represent proportions of distances defined as small, medium, and large, respectively. In addition, they quantify how much the F_1 and F_2 factors present cross-loads. Thus, each distance proportion was obtained through the:

$$d_w = \frac{|\gamma_{w1} - \gamma_{w2}|}{\sqrt{h_w^2}}, \text{ towards } w = 1, 2, 3 \text{ e } 4.$$

The three proportions of the distances between the parametric factor loadings of the F_1 and F_2 factors and defined in each of the six different parametric commonalities h_w^2, were aimed at establishing different degrees of separation of the four standardized variables (Z_1, Z_2, Z_3, and Z_4) in the first two factors.

In addition, it was possible to define, using the absolute magnitudes of the parametric factor loadings, the representation of the standardized variables Z_1 and Z_2 by F_1 and, of Z_3 and Z_4, by F_2, setting at:

$(\gamma_{11} = \gamma_{21} = \gamma_{32} = \gamma_{42}) > (\gamma_{12} = \gamma_{22} = \gamma_{31} = \gamma_{41})$.

Subsequently, the matrix $\mathbf{\Psi}$ (4 × 4) of the parametric variances of the specific ($\psi_1 = \psi_2 = \psi_3 = \psi_4$), was calculated, as follows:

$$\mathbf{\Psi} = \begin{bmatrix} \psi_1 = 1 - h_1^2 & 0 & 0 & 0 \\ 0 & \psi_2 = 1 - h_2^2 & 0 & 0 \\ 0 & 0 & \psi_3 = 1 - h_3^2 & 0 \\ 0 & 0 & 0 & \psi_4 = 1 - h_4^2 \end{bmatrix}.$$

Therefore, 18 matrices (4 × 2) of the parametric factor loadings ($\mathbf{\Gamma}$) and 18 matrices (4 x 4) of the parametric variances of the specific factors ($\mathbf{\Psi}$), were obtained, as shown in Tables 1, 2, 3, 4, 5 and 6, with the parametric eigenvalues obtained through the following formula:

$\lambda_j = \sum_{w=1}^{4} \gamma_{wj}^2$, towards j = 1 e 2.

Table 1. Matrices $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ according to d_w, for $h_w^2 = 0.49$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 0.98$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\mathbf{\Gamma}$	$\begin{bmatrix} 0.69 & 0.12 \\ 0.69 & 0.12 \\ 0.12 & 0.69 \\ 0.12 & 0.69 \end{bmatrix}$	$\begin{bmatrix} 0.628 & 0.31 \\ 0.628 & 0.31 \\ 0.31 & 0.628 \\ 0.31 & 0.628 \end{bmatrix}$	$\begin{bmatrix} 0.56 & 0.42 \\ 0.56 & 0.42 \\ 0.42 & 0.56 \\ 0.42 & 0.56 \end{bmatrix}$
$\mathbf{\Psi}$	$\begin{bmatrix} 0.51 & 0 & 0 & 0 \\ 0 & 0.51 & 0 & 0 \\ 0 & 0 & 0.51 & 0 \\ 0 & 0 & 0 & 0.51 \end{bmatrix}$	$\begin{bmatrix} 0.51 & 0 & 0 & 0 \\ 0 & 0.51 & 0 & 0 \\ 0 & 0 & 0.51 & 0 \\ 0 & 0 & 0 & 0.51 \end{bmatrix}$	$\begin{bmatrix} 0.51 & 0 & 0 & 0 \\ 0 & 0.51 & 0 & 0 \\ 0 & 0 & 0.51 & 0 \\ 0 & 0 & 0 & 0.51 \end{bmatrix}$

Table 2. Matrices $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ according to d_w, for $h_w^2 = 0.56$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 1.13$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\mathbf{\Gamma}$	$\begin{bmatrix} 0.738 & 0.134 \\ 0.738 & 0.134 \\ 0.134 & 0.738 \\ 0.134 & 0.738 \end{bmatrix}$	$\begin{bmatrix} 0.672 & 0.333 \\ 0.672 & 0.333 \\ 0.333 & 0.672 \\ 0.333 & 0.672 \end{bmatrix}$	$\begin{bmatrix} 0.6 & 0.45 \\ 0.6 & 0.45 \\ 0.45 & 0.6 \\ 0.45 & 0.6 \end{bmatrix}$
$\mathbf{\Psi}$	$\begin{bmatrix} 0.44 & 0 & 0 & 0 \\ 0 & 0.44 & 0 & 0 \\ 0 & 0 & 0.44 & 0 \\ 0 & 0 & 0 & 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.44 & 0 & 0 & 0 \\ 0 & 0.44 & 0 & 0 \\ 0 & 0 & 0.44 & 0 \\ 0 & 0 & 0 & 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.44 & 0 & 0 & 0 \\ 0 & 0.44 & 0 & 0 \\ 0 & 0 & 0.44 & 0 \\ 0 & 0 & 0 & 0.44 \end{bmatrix}$

Table 3. Matrices $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ according to d_w, for $h_w^2 = 0.64$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 1.28$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\mathbf{\Gamma}$	$\begin{bmatrix} 0.788 & 0.138 \\ 0.788 & 0.138 \\ 0.138 & 0.788 \\ 0.138 & 0.788 \end{bmatrix}$	$\begin{bmatrix} 0.715 & 0.359 \\ 0.715 & 0.359 \\ 0.359 & 0.715 \\ 0.359 & 0.715 \end{bmatrix}$	$\begin{bmatrix} 0.64 & 0.48 \\ 0.64 & 0.48 \\ 0.48 & 0.64 \\ 0.48 & 0.64 \end{bmatrix}$
$\mathbf{\Psi}$	$\begin{bmatrix} 0.36 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 \\ 0 & 0 & 0.36 & 0 \\ 0 & 0 & 0 & 0.36 \end{bmatrix}$	$\begin{bmatrix} 0.36 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 \\ 0 & 0 & 0.36 & 0 \\ 0 & 0 & 0 & 0.36 \end{bmatrix}$	$\begin{bmatrix} 0.36 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 \\ 0 & 0 & 0.36 & 0 \\ 0 & 0 & 0 & 0.36 \end{bmatrix}$

Table 4. Matrices $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ according to d_w, for $h_w^2 = 0.72$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 1.45$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\mathbf{\Gamma}$	$\begin{bmatrix} 0.837 & 0.148 \\ 0.837 & 0.148 \\ 0.148 & 0.837 \\ 0.148 & 0.837 \end{bmatrix}$	$\begin{bmatrix} 0.76 & 0.38 \\ 0.76 & 0.38 \\ 0.38 & 0.76 \\ 0.38 & 0.76 \end{bmatrix}$	$\begin{bmatrix} 0.68 & 0.51 \\ 0.68 & 0.51 \\ 0.51 & 0.68 \\ 0.51 & 0.68 \end{bmatrix}$
$\mathbf{\Psi}$	$\begin{bmatrix} 0.28 & 0 & 0 & 0 \\ 0 & 0.28 & 0 & 0 \\ 0 & 0 & 0.28 & 0 \\ 0 & 0 & 0 & 0.28 \end{bmatrix}$	$\begin{bmatrix} 0.28 & 0 & 0 & 0 \\ 0 & 0.28 & 0 & 0 \\ 0 & 0 & 0.28 & 0 \\ 0 & 0 & 0 & 0.28 \end{bmatrix}$	$\begin{bmatrix} 0.28 & 0 & 0 & 0 \\ 0 & 0.28 & 0 & 0 \\ 0 & 0 & 0.28 & 0 \\ 0 & 0 & 0 & 0.28 \end{bmatrix}$

Table 5. Matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$ according to d_w, for $h_w^2 = 0.81$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 1.62$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\boldsymbol{\Gamma}$	$\begin{bmatrix} 0.886 & 0.158 \\ 0.886 & 0.158 \\ 0.158 & 0.886 \\ 0.158 & 0.886 \end{bmatrix}$	$\begin{bmatrix} 0.805 & 0.402 \\ 0.805 & 0.402 \\ 0.402 & 0.805 \\ 0.402 & 0.805 \end{bmatrix}$	$\begin{bmatrix} 0.72 & 0.54 \\ 0.72 & 0.54 \\ 0.54 & 0.72 \\ 0.54 & 0.72 \end{bmatrix}$
$\boldsymbol{\Psi}$	$\begin{bmatrix} 0.19 & 0 & 0 & 0 \\ 0 & 0.19 & 0 & 0 \\ 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.19 & 0 & 0 & 0 \\ 0 & 0.19 & 0 & 0 \\ 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.19 & 0 & 0 & 0 \\ 0 & 0.19 & 0 & 0 \\ 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0.19 \end{bmatrix}$

Table 6. Matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$ according to d_w, for $h_w^2 = 0.90$ (w = 1, 2, 3 e 4) and $\lambda_1 = \lambda_2 = 1.81$

Matrix	$d_w = 0.81$	$d_w = 0.45$	$d_w = 0.20$
$\boldsymbol{\Gamma}$	$\begin{bmatrix} 0.935 & 0.168 \\ 0.935 & 0.168 \\ 0.168 & 0.935 \\ 0.168 & 0.935 \end{bmatrix}$	$\begin{bmatrix} 0.852 & 0.42 \\ 0.852 & 0.42 \\ 0.42 & 0.852 \\ 0.42 & 0.852 \end{bmatrix}$	$\begin{bmatrix} 0.76 & 0.57 \\ 0.76 & 0.57 \\ 0.57 & 0.76 \\ 0.57 & 0.76 \end{bmatrix}$
$\boldsymbol{\Psi}$	$\begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$

2.3 Correlations Matrices

According to the orthogonal factorial model, the matrix (4×4) $\boldsymbol{\Sigma}_{\mathbf{Z}} = \boldsymbol{\rho}_{\mathbf{Y}}$ $(\boldsymbol{\rho})$ was obtained by: $\boldsymbol{\Gamma}\boldsymbol{\Gamma}' + \boldsymbol{\Psi} = \boldsymbol{\rho}$.

Thus, from the 18 matrices (4 x 2) of the parametric factor loadings $(\boldsymbol{\Gamma})$ and the 18 matrices (4 x 4) of the parametric variances of the specific factors $(\boldsymbol{\Psi})$, 18 matrices (4 x 4) of parametric correlations were obtained $(\boldsymbol{\rho})$, all of them positive, among the four original variables (Y_1, Y_2, Y_3 e Y_4), as shown in Tables 7, 8, 9, 10, 11 and 12, in which:

$$\boldsymbol{\rho} = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12} & 1 & \rho_{23} & \rho_{24} \\ \rho_{13} & \rho_{23} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix}, \text{ for: } (\rho_{12} = \rho_{34}) > (\rho_{13} = \rho_{14} = \rho_{23} = \rho_{24}).$$

Table 7. Matrix $\boldsymbol{\rho}$ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.49$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 0.98$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.49	0.17	0.17	1	0.49	0.39	0.39	1	0.49	0.47	0.47
	1	0.17	0.17		1	0.39	0.39		1	0.47	0.47
		1	0.49			1	0.49			1	0.49
			1				1				1

Table 8. Matrix $\boldsymbol{\rho}$ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.56$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 1.13$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.56	0.20	0.20	1	0.56	0.45	0.45	1	0.56	0.54	0.54
	1	0.20	0.20		1	0.45	0.45		1	0.54	0.54
		1	0.56			1	0.56			1	0.56
			1				1				1

Table 9. Matrix ρ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.64$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 1.28$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.64	0.22	0.22	1	0.64	0.51	0.51	1	0.64	0.61	0.61
	1	0.22	0.22		1	0.51	0.51		1	0.61	0.61
		1	0.64			1	0.64			1	0.64
			1				1				1

Table 10. Matrix ρ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.72$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 1.45$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.72	0.25	0.25	1	0.72	0.58	0.58	1	0.72	0.70	0.70
	1	0.25	0.25		1	0.58	0.58		1	0.70	0.70
		1	0.72			1	0.72			1	0.72
			1				1				1

Table 11. Matrix ρ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.81$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 1.62$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.81	0.28	0.28	1	0.81	0.65	0.65	1	0.81	0.78	0.78
	1	0.28	0.28		1	0.65	0.65		1	0.78	0.78
		1	0.81			1	0.81			1	0.81
			1				1				1

Table 12. Matrix ρ represented by the elements on the diagonal and above it according to d_w, for $h_w^2 = 0.90$ (w = 1, 2, 3 and 4) and $\lambda_1 = \lambda_2 = 1.81$

$d_w = 0.81$				$d_w = 0.45$				$d_w = 0.20$			
1	0.90	0.31	0.31	1	0.90	0.72	0.72	1	0.90	0.87	0.87
	1	0.31	0.31		1	0.72	0.72		1	0.87	0.87
		1	0.90			1	0.90			1	0.90
			1				1				1

2.4 Principal Method Components

The principal components estimation method was used to perform 36 factor analyses, fixing the estimation of the first two orthogonal factors of each of the 18 matrices (4×4) of parametric correlations (ρ), according to the following approximation:

$\mathbf{R} \cong \hat{\Gamma}\hat{\Gamma}' + \hat{\Psi}$, where:

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ r_{12} & 1 & r_{23} & r_{24} \\ r_{13} & r_{23} & 1 & r_{34} \\ r_{14} & r_{24} & r_{34} & 1 \end{bmatrix} = \text{(4 x 4) sample correlation matrix;}$$

$$\hat{\Gamma} = \begin{bmatrix} \hat{\gamma}_{11} & \hat{\gamma}_{12} \\ \hat{\gamma}_{21} & \hat{\gamma}_{22} \\ \hat{\gamma}_{31} & \hat{\gamma}_{32} \\ \hat{\gamma}_{41} & \hat{\gamma}_{42} \end{bmatrix} = \text{(4 x 2) sample factor loadings;}$$

$$\hat{\mathbf{\Psi}} = \begin{bmatrix} \hat{\psi}_1 & 0 & 0 & 0 \\ 0 & \hat{\psi}_2 & 0 & 0 \\ 0 & 0 & \hat{\psi}_3 & 0 \\ 0 & 0 & 0 & \hat{\psi}_4 \end{bmatrix} = \text{matrix (4 x 4) of sample variances of specific factors.}$$

2.4.1 *Without Rotation*

For the first two factors (F_1 and F_2), the following matrix (4×2) of sample factor loadings was obtained:

$$\hat{\Gamma} = \begin{bmatrix} \sqrt{\hat{\lambda}_1}\hat{e}_{11} & \sqrt{\hat{\lambda}_2}\hat{e}_{12} \\ \sqrt{\hat{\lambda}_1}\hat{e}_{21} & \sqrt{\hat{\lambda}_2}\hat{e}_{22} \\ \sqrt{\hat{\lambda}_1}\hat{e}_{31} & \sqrt{\hat{\lambda}_2}\hat{e}_{32} \\ \sqrt{\hat{\lambda}_1}\hat{e}_{41} & \sqrt{\hat{\lambda}_2}\hat{e}_{42} \end{bmatrix}, \text{ where:}$$

$\hat{e}_{wj}$ = estimate of the coefficient e_{wj} of the normalized eigenvector (4×1) $\mathbf{e_j}$;

$\hat{\gamma}_{jw} = \sqrt{\hat{\lambda}_j}\hat{e}_{wj}$ = estimate of the γ_{wj} factor loading γ_{wj};

$\hat{\lambda}_j = \sum_{w=1}^{4} \hat{\gamma}_{wj}^2$ = estimate of the λ_j eigenfactor; and

$\hat{h}_w^2 = \sum_{j=1}^{k} \hat{\gamma}_{wj}^2$ = estimation of commonality h_w^2 (w = 1, 2, 3 and 4, j = 1 and 2).

Besides the (4 x 2) $\hat{\Gamma}$ matrix, the following (4 x 4) sample matrix was obtained:

$\mathbf{MRes} = \mathbf{R} - (\hat{\Gamma}\hat{\Gamma}' + \hat{\Psi})$, where:

SQRes = tr $[(\mathbf{MRes})^2]$.

With the aid of the *openxlsx* (Schauberger *et al.*, 2019) and *psych* (Revelle, 2020) packages of the R software, 18 factor analyses were performed using the principal component estimation method, without rotation, according to the 18 correlation matrices (ρ) stored in the data files *cor1.xlsx*, *cor2.xlsx*, ..., *cor18.xlsx*, respectively. Each data file, generically represented by *cor_.xlsx*, was composed of five lines and four columns, whose column names arranged in the first line were defined, respectively, by z1, z2, z3, and z4. Thus, the estimation through the principal components method, without rotation, was obtained according to the following *script*:

```
library (openxlsx)
dados = read.xlsx ("cor_.xlsx")
attach (dados)
cor = cbind (z1, z2, z3, z4)
library (psych)
analise.cp = principal (cor, nfactors = 2, rotate = "none")
comu = analise.cp$communality
a = analise.cp$loadings
at = t (a)
gg = a%*%at
varexp = diag (1 - comu)
corest = gg + varexp
autovalor = analise.cp$values
e = cor - gg - varexp
e2 = e%*%e
sqres = tr (e2)
sqres
```

2.4.2 With Rotation

For the first two orthogonal rotated factors (F_1^* and F_2^*) by the varimax criterion, the following matrix (4×2) of rotated and sampling factor loadings was obtained:

Para os dois primeiros fatores rotacionados (F_1^* e F_2^*) ortogonais pelo critério varimax, foi obtida a seguinte matriz (4×2) das cargas fatoriais rotacionadas e amostrais:

$$\widehat{\Gamma}^* = \begin{bmatrix} \sqrt{\hat{\lambda}_1^*}\hat{e}_{11}^* & \sqrt{\hat{\lambda}_2^*}\hat{e}_{12}^* \\ \sqrt{\hat{\lambda}_1^*}\hat{e}_{21}^* & \sqrt{\hat{\lambda}_2^*}\hat{e}_{22}^* \\ \sqrt{\hat{\lambda}_1^*}\hat{e}_{31}^* & \sqrt{\hat{\lambda}_2^*}\hat{e}_{32}^* \\ \sqrt{\hat{\lambda}_1^*}\hat{e}_{41}^* & \sqrt{\hat{\lambda}_2^*}\hat{e}_{42}^* \end{bmatrix}, \text{ where:}$$

$\hat{e}_{wj}^*$ = estimate of the rotated coefficient and e_{wj}^* of the rotated normalized eigenvector (4 x 1) $\mathbf{e}_{wj}^*$;

$\hat{\gamma}_{wj}^* = \sqrt{\hat{\lambda}_j}\hat{e}_{wj}^*$ = estimate of the rotated factor loading γ_{wj}^*; and

$\hat{\lambda}_j^* = \sum_{w=1}^{4} \hat{\gamma}_{wj}^{*2}$ = estimate of the rotated eigenvalue λ_j^* (w = 1, 2, 3 and 4 and j = 1 and 2) .

On the other hand, the *script* used for the estimation using the principal components method, with varimax rotation and according to the 18 matrices (4×4) of parametric correlations ($\boldsymbol{\rho}$), has suffered only a change in the command line, as follows:

```
...
analise.cp = principal (cor, nfactors = 2, rotate = "varimax")
```

2.5 Statistical Analysis

In order to evaluate the effects of the parametric commonality (h_w^2) and the proportion of the distance between the parametric factor loadings of the F_1 and F_2 (d_w)on the qualities of the estimates of the commonalities of the standardized variables Z_1 (h_1^2), Z_2 (h_2^2), Z_3 (h_3^2) and Z_4 (h_4^2), as well as the estimation of the matrix (4×4) $\boldsymbol{\Psi}$, a response surface analysis of a 6×3 factorial experiments was carried out under a completely randomized design (DIC) with no repetition. So, three evaluated variables were obtained:

$\Delta h^2 = \frac{1}{4}\left(\sum_{w=1}^{4} \frac{|h_w^2 - \hat{h}_w^2|}{h_w^2}\right)$;

SQRes = tr $\{[\mathbf{R} - (\widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\Gamma}}' + \widehat{\boldsymbol{\Psi}})]^2\}$ = tr $[(\mathbf{MRes})^2]$; and

$\Delta\Gamma = \frac{1}{8}\left(\sum_{w=1}^{4}\sum_{j=1}^{2} \frac{|\gamma_{wj} - \hat{\gamma}_{jw}|}{\gamma_{wj}}\right)$.

In this case, the following largest first-order model was adopted:

$y = \beta_0 + \beta_1 h + \beta_2 d + \beta_3 hd + \varepsilon$, where:

y = observed value of the evaluated variable (Δh^2, SQRes e $\Delta\Gamma$);

h = parametric commonality (h_w^2) (0.49; 0.56; 0.64; 0.72; 0.81; 0.90);

d = proportion of the distance between the parametric factor loadings of factors F_1 and F_2 (d_w) (0.20; 0.45; and 0.81); and

$\varepsilon \sim N(0; \sigma^2)$.

In addition, to evaluate the effects of the parametric commonality (h_w^2), the proportion of the distance between the parametric factor loadings of F_1 and F_2 (d_w) and the varimax rotation on the qualities of the estimates of the first two parametric eigenvalues and the eight parametric factor loadings of the first two factors, a response surface analysis of a $6 \times 3 \times 2$ factorial experiments was performed under DIC without repetition. So, two other evaluated variables were obtained:

$\Delta\lambda_1 = \frac{|\lambda_1 - \hat{\lambda}_1|}{\lambda_1}$; and

$\Delta\lambda_2 = \frac{|\lambda_2 - \hat{\lambda}_2|}{\lambda_2}$.

And in this case, the following largest first-order model was adopted:

$y = \beta_0 + \beta_1 h + \beta_2 d + \beta_3 r + \beta_4 hd + \beta_5 hr + \beta_6 dr + \varepsilon$, where:

y = observed value of the evaluated variable ($\Delta\lambda_1$ and $\Delta\lambda_2$);

h = parametric commonality (h_w^2) (0.49; 0.56; 0.64; 0.72; 0.81; and 0.90);

d = proportion of the distance between the parametric factor loadings of factors F_1 and F_2 (d_w) (0.20; 0.45; and 0.81);

r = varimax rotation (r = 0 = without and r = 1 = with); and

$\varepsilon \sim N(0; \sigma^2)$.

In both models, the non-significant coefficients were removed from the model, one by one, and starting with the interactions, according to Student's t-test at 5% significance. Finally, the adjusted model was composed only of the significant effects, except the linear effects that showed significant interactions.

Statistical analyses were performed using the R software (version 3.3.1) and Microsoft Excel 2016.

3. Results and Discussion

3.1 Effects of Commonality and Proportion of Distance

The evaluated variables, Δh^2 and SQRes, presented decreases in their values due to the increase ($P < 0.05$) of the parametric commonality (h_w^2), but were not influenced ($P > 0.05$) by the proportion of the distance between the parametric factor loadings of the first two factors (d_w). On the other hand, $\Delta\Gamma$ decreased due to the reduction ($P < 0.05$) in dw and without interference ($P > 0.05$) of h_w^2 (Table 13).

Table 13. Estimates of Δh^2, SQRes, and $\Delta\Gamma$ as a function of parametric commonality (h_w^2) and the proportion of the distance between the parametric factor loadings of the F_1 and F_2 (d_w)

Variable	Model Adjusted	R^2
Δh^2	$1.02331 - 1.11124^*h$	0.97
SQRes	$0.53537 - 0.60782^*h$	0.96
$\Delta\Gamma$	$0.0535 + 2.4886^*d$	0.82

*: significant by Student's t-test ($P < 0.05$); $h = h_w^2$; $0.49 \le h \le 0.90$; $d = d_w$; $0.20 \le d \le 0.81$.

As Δh^2 was inversely related to h_w^2, the error in estimating the parametric commonality of the standardized variable Z_w (w = 1, 2, 3 and 4) due to the principal components method increased as it decreased. Therefore, the greater the parametric commonality of a variable, the better its estimate and, consequently, the better its interpretation based on factor analysis.

In this work, the estimation error based on $h_w^2 - \hat{h}_w^2$ was negative, given that the estimates of h_w^2 equal to 0.49; 0.56; 0.64; 0.72; 0.81; and 0.90 were, respectively, equal to: 0.75; 0.78; 0.82; 0.86; 0.91 and 0.95. This means that the principal components estimation method overestimates the parametric commonality.

In addition, SQRes also helps in assessing the goodness of fit of the orthogonal factorial model. The greater the h_w^2 of the standardized variable Z_w, the smaller its specific variance and, consequently, the smaller the difference between $\boldsymbol{\rho}$ and $\boldsymbol{\Gamma\Gamma'} + \boldsymbol{\Psi}$, as the diagonal of the matrix $\boldsymbol{\Psi}$ is equivalent to $1 - h_w^2$ (Ferreira, 2018). Thus, as the SQRes decreases as the h_w^2 increases, the better the fit quality of the factorial model when higher estimates of h_w^2 continually occur.

Mingoti (2013) and Cruz *et al.* (2014) reported values greater than 0.64 as acceptable estimates of h_w^2. On the other hand, Hair *et al.* (2014) stated that variables with commonalities below 0.50 do not have sufficient explanations by the factor analysis. Therefore, such conclusions were supported, given that as the parametric commonality increased, its estimation error decreased.

On the other hand, contrary to expectation, $\Delta\Gamma$ decreased only as the d_w was reduced. It was first believed that $\Delta\Gamma$ would increase as a function of increases in h_w^2 and d_w as the increases in the latter would characterize greater relationships between the variables grouped in the respective factors. The greater the proportion of the distance between the parametric factor loadings of factors F_1 and F_2, the further from each other will be the parametric correlations of Y_w and F_j, thus facilitating the grouping of variables in the respective factors. Therefore, the lack of this result suggested that the principal components estimation method was not adequate, in relation to the estimates of parametric factorial loads when applied without the aid of rotation.

In all sample matrices of factor loadings ($\hat{\boldsymbol{\Gamma}}$), regardless of h_w^2 and d_w, the principal components estimation method provided the same absolute estimates of the parametric factor loadings of F_1 and F_2, separately. Also, as the absolute estimates of the parametric factor loadings of F_1 were greater than those of F_2, the standardized variables Z_1, Z_2, Z_3 and Z_4 were grouped in the F_1 factor. This would lead to an incorrect interpretation that all four standardized variables present positive correlations with each other and that they can be represented by only a single factor, even for cases with $h_w^2 \geq 0.64$ (Table 14). Parametrically, Z_1 and Z_1 are associated with the F_1 factor, and Z_3 and Z_4 are not correlated with Z_1 and Z_2, with F2.

Em todas as matrizes amostrais de cargas fatoriais ($\hat{\Gamma}$), independentemente de h_w^2 e d_w, o método de estimação dos componentes principais proporcionou as mesmas estimativas absolutas das cargas fatoriais paramétricas de F_1 e de F_2, separadamente. E como as estimativas absolutas das cargas fatoriais paramétricas de F_1 foram maiores do que as de F_2, as variáveis padronizadas Z_1, Z_2, Z_3 e Z_4 foram agrupadas no fator F_1. Isso induziria em uma interpretação incorreta de que todas as quatro variáveis padronizadas apresentam correlações positivas entre si e que podem ser representadas por apenas um único fator, mesmo para os casos com $h_w^2 \geq 0{,}64$ (Table 14). De forma paramétrica, Z_1 e Z_2 associam-se ao fator F_1 e, Z_3 e Z_4, não correlacionadas com Z_1 e Z_2, ao F_2.

Table 14. $\hat{\Gamma}$ and $\hat{\Gamma}^*$ matrices according to d_w and h_w^2 (w = 1, 2, 3 and 4)

h_w^2	$d_w = 0.81$		$d_w = 0.45$		$d_w = 0.20$	
	$\hat{\Gamma}$	$\hat{\Gamma}^*$	$\hat{\Gamma}$	$\hat{\Gamma}^*$	$\hat{\Gamma}$	$\hat{\Gamma}^*$
0.49	$\begin{bmatrix}\mathbf{0.68} & 0.54 \\ \mathbf{0.68} & 0.54 \\ \mathbf{0.68} & -0.54 \\ \mathbf{0.68} & -0.54\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.80} & -0.32 \\ \mathbf{0.80} & -0.32 \\ 0.49 & \mathbf{0.71} \\ 0.49 & \mathbf{0.71}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.75} & -0.42 \\ \mathbf{0.75} & -0.42 \\ \mathbf{0.75} & 0.42 \\ \mathbf{0.75} & 0.42\end{bmatrix}$	$\begin{bmatrix}0.26 & \mathbf{0.82} \\ 0.26 & \mathbf{0.82} \\ \mathbf{0.84} & 0.21 \\ \mathbf{0.84} & 0.21\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.78} & 0.37 \\ \mathbf{0.78} & 0.37 \\ \mathbf{0.78} & -0.37 \\ \mathbf{0.78} & -0.37\end{bmatrix}$	$\begin{bmatrix}0.58 & \mathbf{0.64} \\ 0.58 & \mathbf{0.64} \\ \mathbf{0.86} & -0.05 \\ \mathbf{0.86} & -0.05\end{bmatrix}$
0.56	$\begin{bmatrix}\mathbf{0.70} & -0.54 \\ \mathbf{0.70} & -0.54 \\ \mathbf{0.70} & 0.54 \\ \mathbf{0.70} & 0.54\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.71} & -0.53 \\ \mathbf{0.71} & -0.53 \\ \mathbf{0.69} & 0.55 \\ \mathbf{0.69} & 0.55\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.78} & -0.41 \\ \mathbf{0.78} & -0.41 \\ \mathbf{0.78} & 0.41 \\ \mathbf{0.78} & 0.41\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.88} & -0.02 \\ \mathbf{0.88} & -0.02 \\ 0.53 & \mathbf{0.71} \\ 0.53 & \mathbf{0.71}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.81} & 0.35 \\ \mathbf{0.81} & 0.35 \\ \mathbf{0.81} & -0.35 \\ \mathbf{0.81} & -0.35\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.81} & 0.35 \\ \mathbf{0.81} & 0.35 \\ \mathbf{0.82} & -0.34 \\ \mathbf{0.82} & -0.34\end{bmatrix}$
0.64	$\begin{bmatrix}\mathbf{0.72} & -0.55 \\ \mathbf{0.72} & -0.55 \\ \mathbf{0.72} & 0.55 \\ \mathbf{0.72} & 0.55\end{bmatrix}$	$\begin{bmatrix}0.62 & \mathbf{0.67} \\ 0.62 & \mathbf{0.67} \\ \mathbf{0.80} & -0.42 \\ \mathbf{0.80} & -0.42\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.82} & -0.39 \\ \mathbf{0.82} & -0.39 \\ \mathbf{0.82} & 0.39 \\ \mathbf{0.82} & 0.39\end{bmatrix}$	$\begin{bmatrix}0.58 & \mathbf{0.70} \\ 0.58 & \mathbf{0.70} \\ \mathbf{0.91} & -0.02 \\ \mathbf{0.91} & -0.02\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.85} & -0.32 \\ \mathbf{0.85} & -0.32 \\ \mathbf{0.85} & 0.32 \\ \mathbf{0.85} & 0.32\end{bmatrix}$	$\begin{bmatrix}0.43 & \mathbf{0.80} \\ 0.43 & \mathbf{0.80} \\ \mathbf{0.85} & 0.32 \\ \mathbf{0.85} & 0.32\end{bmatrix}$
0.72	$\begin{bmatrix}\mathbf{0.75} & -0.55 \\ \mathbf{0.75} & -0.55 \\ \mathbf{0.75} & 0.55 \\ \mathbf{0.75} & 0.55\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.78} & -0.51 \\ \mathbf{0.78} & -0.51 \\ \mathbf{0.71} & 0.60 \\ \mathbf{0.71} & 0.60\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.85} & -0.38 \\ \mathbf{0.85} & -0.38 \\ \mathbf{0.85} & 0.38 \\ \mathbf{0.85} & 0.38\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.92} & -0.13 \\ \mathbf{0.92} & -0.13 \\ \mathbf{0.71} & 0.59 \\ \mathbf{0.71} & 0.59\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.88} & -0.29 \\ \mathbf{0.88} & -0.29 \\ \mathbf{0.88} & 0.29 \\ \mathbf{0.88} & 0.29\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.91} & -0.20 \\ \mathbf{0.91} & -0.20 \\ \mathbf{0.85} & 0.38 \\ \mathbf{0.85} & 0.38\end{bmatrix}$
0.81	$\begin{bmatrix}\mathbf{0.77} & -0.56 \\ \mathbf{0.77} & -0.56 \\ \mathbf{0.77} & 0.56 \\ \mathbf{0.77} & 0.56\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.91} & -0.29 \\ \mathbf{0.91} & -0.29 \\ 0.55 & \mathbf{0.77} \\ 0.55 & \mathbf{0.77}\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.88} & -0.36 \\ \mathbf{0.88} & -0.36 \\ \mathbf{0.88} & 0.36 \\ \mathbf{0.88} & 0.36\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.89} & -0.33 \\ \mathbf{0.89} & -0.33 \\ \mathbf{0.87} & 0.39 \\ \mathbf{0.87} & 0.39\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.92} & 0.25 \\ \mathbf{0.92} & 0.25 \\ \mathbf{0.92} & -0.25 \\ \mathbf{0.92} & -0.25\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.95} & 0 \\ \mathbf{0.95} & 0 \\ \mathbf{0.82} & 0.49 \\ \mathbf{0.82} & 0.49\end{bmatrix}$
0.90	$\begin{bmatrix}\mathbf{0.80} & -0.56 \\ \mathbf{0.80} & -0.56 \\ \mathbf{0.80} & 0.56 \\ \mathbf{0.80} & 0.56\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.83} & -0.51 \\ \mathbf{0.83} & -0.51 \\ \mathbf{0.76} & 0.62 \\ \mathbf{0.76} & 0.62\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.91} & -0.34 \\ \mathbf{0.91} & -0.34 \\ \mathbf{0.91} & 0.34 \\ \mathbf{0.91} & 0.34\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.93} & -0.31 \\ \mathbf{0.93} & -0.31 \\ \mathbf{0.90} & 0.38 \\ \mathbf{0.90} & 0.38\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.95} & 0.21 \\ \mathbf{0.95} & 0.21 \\ \mathbf{0.95} & -0.21 \\ \mathbf{0.95} & -0.21\end{bmatrix}$	$\begin{bmatrix}\mathbf{0.96} & -0.20 \\ \mathbf{0.96} & -0.20 \\ \mathbf{0.95} & 0.22 \\ \mathbf{0.95} & 0.22\end{bmatrix}$

On the other hand, for 44.4% of the sample matrices of rotated factor loadings ($\hat{\Gamma}^*$), the method of estimating the principal components enabled to separate Z_1 and Z_2 from Z_3 e Z_4 in the first two factors, however, without an explanation pattern (Table 14). This meant that the varimax rotation was not able, at first, to improve the estimates of the parametric factor loadings obtained according to the method of principal components. Ideally, it is desired that each variable has a high factor loading, in the module, in a single factor, therefore should present small or moderate absolute loads in the other factors (Johnson; Wichern, 2002).

According to Rogers (2022), the principal components estimation method overestimates the factor loads and the variance explained by the factors, which is in line with the results obtained in this experiment. In addition, Hauck Filho and Valentini (2020) stated that when performing an EFA using this estimation method, a typical EFA is not truly being developed.

3.2 Effects of Commonality, Distance Proportion, and Varimax Rotation

The evaluated variable, $\Delta\lambda_1$ had their values decreased due to the increases ($P < 0.05$) in parametric commonality (h_w^2) and in the proportion of the distance between the parametric factor loadings of factors F_1 and F_2 (d_w) and varimax rotation ($P < 0.05$). On the other hand, $\Delta\lambda_2$ showed the lowest values when the lowest values of h_w^2 and d_w were combined (Table 15).

Table 15. - Estimates of $\Delta\lambda_1$ and $\Delta\lambda_2$ as a function of the parametric commonality (h_w^2), the proportion of the distance between the parametric factor loadings of the F_1 and F_2 factors (d_w), and the varimax rotation

Variable	Model Adjusted	R^2
$\Delta\lambda_1$	1.88082 – 0.72695*h – 0.86473*d – 0.16009*r	0.71
$\Delta\lambda_2$	–0.4207 + 1.7789*h + 0.3836d – 1.6882*hd	0.80

*: significant by Student's t-test ($P < 0.05$); $h = h_w^2$; $0.49 \le h \le 0.90$; $d = d_w$; $0.20 \le d \le 0.81$; r = varimax rotation; r = 0 = without and r = 1 = with.

As $\Delta\lambda_1$ was inversely related to h_w^2 and d_w, the estimation error of the first parametric eigenvalue due to the principal components method increased as they decreased. Therefore, the greater the parametric commonality of a variable and the proportion of the distance between the parametric factor loadings of the same variable in different factors, the better its estimate. In addition, to improve it a little more, it is recommended to use the varimax rotation.

In this work, the estimation error based on $\lambda_1 - \hat{\lambda}_1$ was negative (Table 16). This means that the principal components method overestimates the first parametric eigenvalue (λ_1).

Table 16. Estimates of λ_1, without ($\hat{\lambda}_1$) and with ($\hat{\lambda}_1^*$) the varimax rotation according to d_w and λ_1

	$d_w = 0.81$		$d_w = 0.45$		$d_w = 0.20$	
λ_1	$\hat{\lambda}_1$	$\hat{\lambda}_1^*$	$\hat{\lambda}_1$	$\hat{\lambda}_1^*$	$\hat{\lambda}_1$	$\hat{\lambda}_1^*$
0.98	1.82	1.77	2.27	1.53	2.43	2.16
1.13	1.96	1.96	2.46	2.12	2.64	2.64
1.28	2.07	2.05	2.67	2.31	2.87	1.80
1.45	2.22	2.22	2.88	2.71	3.11	3.08
1.62	2.37	2.26	3.11	3.10	3.36	3.14
1.81	2.53	2.52	3.33	3.33	3.63	3.63

As there was an overestimation of λ_1, the estimate of the second parametric eigenvalue (λ_2) was also compromised and, particularly underestimated (Table 17).

Table 17. Estimates of λ_2, without ($\hat{\lambda}_2$) and with ($\hat{\lambda}_2^*$) the varimax rotation according to d_w and λ_2

	$d_w = 0.81$		$d_w = 0.45$		$d_w = 0.20$	
λ_2	$\hat{\lambda}_2$	$\hat{\lambda}_2^*$	$\hat{\lambda}_2$	$\hat{\lambda}_2^*$	$\hat{\lambda}_2$	$\hat{\lambda}_2^*$
0.98	1.16	1.22	0.71	1.45	0.55	0.82
1.13	1.17	1.17	0.67	1.01	0.48	0.48
1.28	1.21	1.23	0.62	0.97	0.41	1.48
1.45	1.23	1.23	0.57	0.74	0.34	0.36
1.62	1.25	1.36	0.52	0.52	0.25	0.47
1.81	1.27	1.28	0.47	0.48	0.17	0.17

According to Kaiser's (1958) criterion for choosing the number of orthogonal factors ($\lambda_j \ge 1$), the first two factors, with or without varimax rotation, would be correctly used in the factor analysis only for d_w = 0.81 (Tables 16 and 17), reinforcing the λ_1 overestimation. Therefore, the principal components estimation method seems to have a characteristic of overestimating the variability present in the correlation matrix in the first eigenvalue.

In general, despite improving the interpretation of the factors and the estimates of the parametric eigenvalues, the varimax rotation was not sufficient to fully enable the use of the principal components estimation method.

Corroborating the results obtained, Lloret *et al.* (2017) stated that the combination of the principal components estimation method with varimax rotation is probably the worst way to develop an EFA.

4. Conclusions

The principal components estimation method used in the orthogonal factor analysis does not provide adequate estimates of its parameters, even when performed under parametric commonalities greater than 0.64.

Varimax rotation poorly improves the quality of the EFA estimates. On the other hand, considering that the principal components estimation method does not provide reliable results, the varimax rotation associated with this method would not be valid.

Therefore, the use of varimax rotation associated with the principal components estimation method for performing the EFA is not recommended, as they return results that do not represent the original data and, therefore, promote interpretations.

Conflicts of Interest

The authors have no conflicts of interest to declare.

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