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ARTICLE

Reliability characteristics analysis of Ready-Mix Cement plant under classical and bayesian inferential framework: A Comparative Analysis[1]

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Abstract

This study introduces a novel stochastic model for assessing the reliability characteristics of a Ready-Mix Cement (RMC) plant. The model employs both classical and Bayesian statistical frameworks. The RMC plant comprises five key components: the rolling belt unit, cement & fly ash storage unit, mixing drum unit, controller unit, and all electric component and motors units. Weibull distribution has been used to simulate failure and repair timeframes, and all time-dependent random variables are treated as statistically independent which allows to assess mean time to system failure, steady-state availability, and busy period. Different scale parameters for each component and a common shape parameter have been used in the model. The investigation is facilitated by the semi-Markov approach and the regeneration point technique, presuming a fully functional repair facility for routine maintenance and repairs. Electronic components and motor have provisions for inspection, but the system is assumed to include provisions for preventative maintenance. Expert repair services are presumed to be instantly available due to component wear and tears and non-repairable electrical elements. To highlight the model's significance, a Monte Carlo simulation study is conducted, offering a comparative analysis of mean time to system failure (MTSF), availability, and profit functions across traditional, classical, and Bayesian approaches. The results emphasize the effectiveness of the proposed model in optimizing reliability assessments for RMC plants.

Keywords: RMC plant, Semi-Markovian Approach, Classical Estimation, Bayesian Estimation, Regenerative Point Technique.

1. Introduction

In last few decades, the complexity of industrial and mechanical systems has increased significantly due to rising demand and advancement in technology. These businesses produce goods and services that are essential to our daily lives. The Ready-Mix Concrete (RMC) plants significantly contribute to the rapid construction of infrastructure across numerous nations. Recently system designers have focused on the availability, mean time to failure, and overall performance of RMC plants, which has led to a more thorough analysis of their efficacy. The performance of RMC plants is significantly impacted by various subsystems. Among these, preventive maintenance of the whole system as well as the rolling belt and electrical components play key roles in determining overall efficiency. Therefore, assessing the reliability characteristics of these components is essential for evaluating the performance of the entire plant.

Previous researchers have employed a range of approaches, such as Fault Tree Analysis, Failure Mode and Effects Analysis, the Markov technique, and Reliability Block Diagrams, to evaluate reliability under various failure distribution scenarios. The Weibull distribution, first presented by Weibull in 1951, is one distribution that is frequently used to assess the reliability of industrial systems. This distribution's versatility makes it very useful for life testing. Furthermore, adding spare parts is a tried-and-true method of raising the reliability of these kinds of systems. A model for evaluating the availability function's confidence intervals for systems with Weibull-distributed operation times was presented by Master's *et al.* (1992). A stochastic model considering arbitrary failure rates and common cause failures was introduced by Dhillon & Anuda (1993). Component redundancy in non-repairable systems was optimized by Coit (2001). Yadavalli *et al.* (2005) performed a Bayesian analysis on a two-unit system susceptible to common cause shock failures. While Chien *et al.* (2006) demonstrated asymptotic confidence bounds for a repairable system with defective service facilities. Hsu *et al.* (2009) investigated asymptotic and Bayesian estimating techniques for repairable systems, especially regarding incomplete coverage and reboot procedures.

Gupta *et al.* (2013) used the Weibull distribution to simulate failure and repair rates in a thorough cost analysis of non-identical unit systems. Moreover, Singh *et al.* (2013) offered statistical inferences for time-dependent dynamical systems and Chaturvedi *et al.* (2014) created a strong Bayesian framework for Weibull distribution analysis. Furthermore, Kishan & Jain (2014) examined reliability measures using parameter estimation in parallel unit systems under the assumption that all time-dependent random variables have a shared Weibull distribution with a common shape parameter.

A reliability analysis of non-identical unit systems in a fuzzy environment was carried out by Liu *et al.* (2014). Furthermore, the consequence of cold and hot standby redundancy on thermal power plant availability was examined by Kumar *et al.* (2014). Using the Weibull distribution, Kumar & Saini (2016) created a stochastic model for single-unit systems to measure the benefits of preventive maintenance. Phase time distribution was used by Dongliang *et al.* (2016) to estimate the reliability of non-identical unit systems. Additionally, Kumar *et al.* (2016a, 2016b, 2018) investigated the effects of various preventive maintenance techniques and priority strategies on systems with Weibull-distributed random variables. Semaan (2016) used simulation and queuing theory as two stochastic modelling techniques to examine a ready-mix plant's production processes. The simulation model used Monte Carlo simulation techniques to assess performance measures, whereas the queuing model used queuing techniques in conjunction with a Markovian chain framework.

In their study, Dey *et al.* (2017) expanded the generalized exponential distribution and demonstrated how it may be used to analyse ozone data. A stochastic model with waiting time considerations for parallel systems was presented by Chopra & Ram (2017). In the presence of outliers, Gupta & Singh (2017) analysed the Weibull distribution using both conventional and Bayesian methods. The value of Bayesian statistics was highlighted by Han *et al.* (2018) in several different study areas. Pundir *et al.* (2018) created a stochastic framework that includes priority repair disciplines for parallel systems with non-identical components. Reliability models were put out by Kumar & Kadyan (2018, 2019) to assess industrial system performance using extra variables. Furthermore, under random censoring, Kumar & Kumar (2019) computed several statistical features of the inverse Weibull distribution.

Using a statistical-fuzzy technique, Skrzypczak *et al.* (2020) proposed a process for evaluating the quality of ready-mix concrete. Applying statistical and fuzzy theories to resolve information and random uncertainties, their study focused on concrete quality control and classification. To assess the effects of inspection and degradation, Saini and Kumar (2020) created a stochastic model for single-unit systems operating under unusual environmental settings. Pundir *et al.* (2020) examined the effects of prior knowledge on standby system reliability estimation. Reliability, Availability, and Maintainability (RAM) analysis was carried out by Kumar *et al.* (2020) in order to improve the soft water supply and treatment plant's operating performance. Within a Bayesian framework, Patawa *et al.* (2021) made several deductions about reliability measures for non-identical systems with waiting time and standby redundancy. Additionally, Rathi *et al.* (2022) created a model that uses Markov processes and redundancies to increase reliability. Sengar & Ram (2022) performed a performance and reliability analysis of a complicated production system with an inspection facility using copula approach. Kalili *et al.* (2023) carried out the study to examine effects of the planned preventive maintenance on batch processing industries.

Although significant research has been done on the reliability evaluation of industrial systems, most of it has concentrated on modelling, steady-state availability, MTSF and performance evaluation, frequently presuming constant component failure and repair rates. Still, not sufficient research has been done on parameter estimate, especially when it comes to Ready-Mix Concrete (RMC) facilities. A stochastic model for RMC factory with five components is presented in this study using the Weibull distribution for failure and repair rates with different scale factors and a common shape parameter. An analysis of the stochastic model through simulation is executed in order to extract significant findings.

A semi-Markovian method and the regenerative point technique are applied to generate a number of system reliability measures that are useful to plant designers and maintenance managers. These measures include the mean sojourn time related with distinct regenerative states, the steady-state transition probabilities associated with several states, and the real and estimated values of the Mean Time to System Failure (MTSF) for the RMC plant. Furthermore, the model offers true and estimated values for the RMC plant's steady-state availability and profit.

Both conventional and Bayesian frameworks are used to estimate the parameters of the associated distributions, considering the random behavior of component lifetimes in the RMC plant. Since posterior densities are difficult to replicate directly, random samples are created from these posterior densities using the Metropolis-Hastings algorithm, which is a component of the Markov Chain Monte Carlo (MCMC) process. Furthermore, the statistical values of reliability measures under classical and Bayesian techniques are derived using the Monte Carlo simulation technique. The Mean Square Error (MSE), availability, MTSF, profit and confidence interval length are all assessed in the classical framework. The width of the highest posterior density and the posterior MSE are calculated in the Bayesian framework. Numerical findings and figures are used to give a comparative comparison, highlighting the importance of this work. Starting with this current introduction, the manuscript is divided into five sections. The system description and notations are defined in Section 2. The resulting reliability measures are shown in Section 3, and the parameter estimation in the classical and Bayesian frameworks is covered in Section 4. Lastly, conclusions and future scope of the work are provided in Section 5.

2. System Explanation and Notations

The system explanation of RMC plant and notation applied for model development are given below.

2.1 Notations

S_i: i^{th} state of the RMC plant

Φ_i (i = 1, 2, 3, 4,5, 6, 7, 8): Scale parameter of failure time distribution for i^{th} unit

ψ_i (i = 1, 2, 3, 4,5, 6, 7): Scale parameter of repair time distribution for i^{th} unit

Φ_0: Scale parameter of maximum operational time distribution

ψ_0: Scale parameter of preventive maintenance time distribution

η: Shape parameter of failure/repair time distribution of each unit

$f_i(t)$: Failure rate of i^{th} unit where $f_i(t) = \Phi_i \eta t^{\eta-1} e^{\Phi_i t^{\eta}}$, $\Phi_i > 0, t > 0$

$g_i(t)$: Repair rate of i^{th} unit where $g_i(t) = \psi_i \eta t^{\eta-1} e^{\psi_i t^{\eta}}$, $\psi_i > 0, t > 0$

$f_0(t)$: Maximum operational time

$g_0(t)$: Preventive maintenance rate

$q_{ij}(t)/Q_{ij}(t)$: Pdf and cdf of one step or direct transition time from $S_i \in E$ to $S_j \in E$.

$p_{ij}(t)$: Steady state transition probability from S_i to S_j such that, $p_{ij}(t) = \lim_{t\to\infty} Q_{ij}(t)$.

$Z_i(t)$: Probability that system sojourns in state S_i up to time t.

μ_i: Mean sojourn time in state S_i i.e., $\mu_i = \int_0^\infty P(T_i > t)$.

$R_i(t)$: Reliability of the system at time t when systems start from $S_i \in E$.

$A_i(t)$: Probability that the system will be operative in state $S_i \in E$ at epoch t

$B_i(t)$: Probability that the repairman will be busy in state $S_i \in E$ at epoch t

$\boldsymbol{P_i(t)}$:	Profit incurred by the system during interval (0, t).
**:	Symbol for Laplace Transform of a function i.e., $Q_{ij}^{**}(s) = \int_0^\infty q_{ij}(t)e^{-st}dt$
•:	Regenerative point
$\mathbf{U_0}$:	Rolling Belt unit (U) is operative
$\mathbf{V_0}$:	Cement & Fly ash storage unit (V) is operative
$\mathbf{W_0}$:	Mixing Drum unit (W) is operative
$\mathbf{X_0}$:	Controller unit (X) is operative
$\mathbf{Y_0}$:	All Electric component and motors units (Y) is operative
$\mathbf{u_r}$:	Rolling Belt unit (U) is in non-operative mode and under repair
$\mathbf{v_r}$:	Cement & Fly ash storage unit (V) is in non-operative mode and under repair
$\mathbf{w_r}$:	Mixing Drum unit (W) is in non-operative mode and under repair
$\mathbf{x_r}$:	Controller unit (X) is in non-operative mode and under repair
$\mathbf{y_i}$:	All Electric component and motors units (Y) are under inspection mode
$\mathbf{y_{ro}}$:	All Electric component and motors units (Y) are in non-operative mode and under repair
$\mathbf{y_{rex}}$:	All Electric component and motors units (Y) are in non-operative mode and under repair by expert
$\mathbf{y_{rpl}}$:	All Electric component and motors units (Y) are in non-operative mode and not repairable so replaced by a new one

2.2 System Explanation

The rolling belt unit, cement & fly ash storage unit, mixing drum unit, controller unit, and all electric components and motors units are the five main parts of the RMC plant. Preventive maintenance is scheduled for the entire system, and routine inspection are carried out on the electric component and motor units. Due to the complexity of these electrical components, two types of repair services are available: standard repair and professional repair. Furthermore, it is presumed that a component will be replaced with a new one if it is determined to be irreparable. It is assumed that every repair is flawless and that there hasn't been any component degradation. It is believed that the rates of failure and repair are statistically independent, and that no two failures can happen at the same time. No component has redundancy, but it is expected that a failing unit will be repaired. The Semi-Markovian approach and the regenerating point technique are used to evaluate the RMC plant's reliability features. A stochastic model is proposed, and expressions for several reliability measures are obtained. In addition, both classical and Bayesian inferential frameworks are used for parameter estimation. Figure 1 displays the suggested stochastic model's state transition diagram.

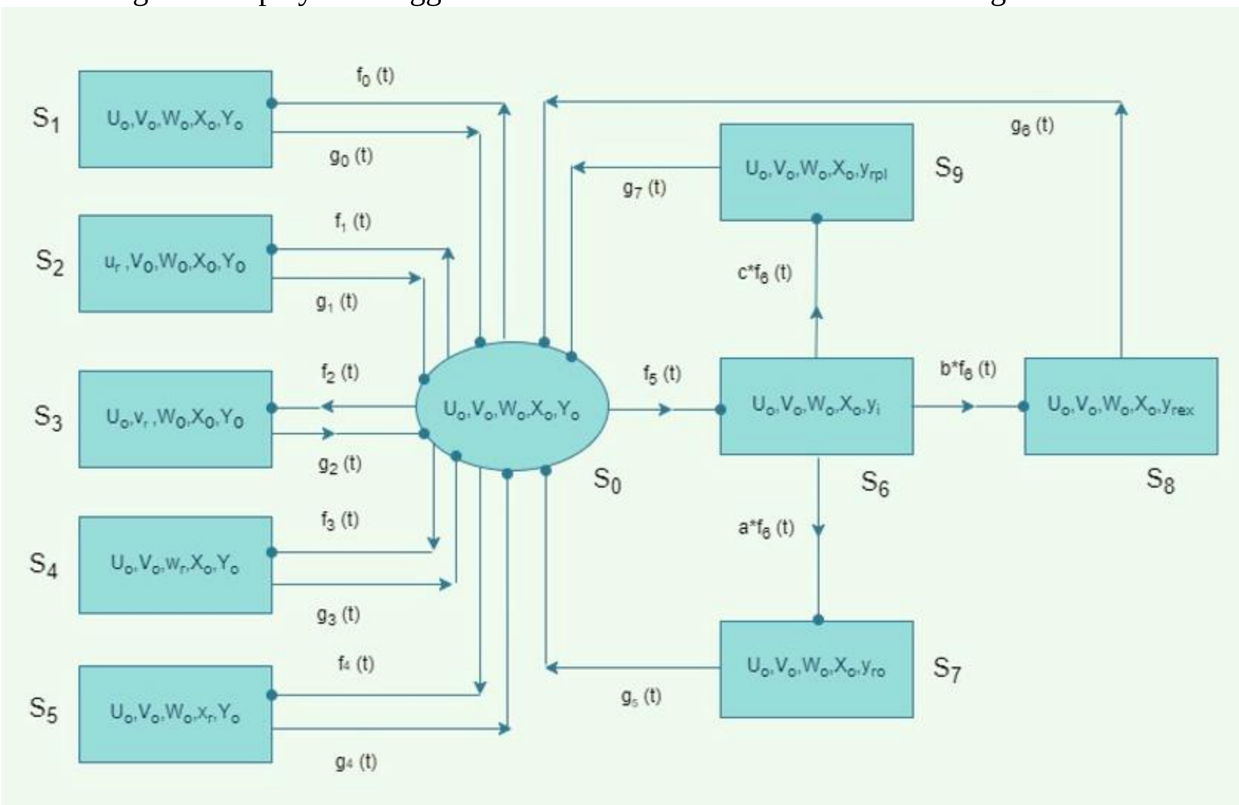

Figure 1. State Transition Flow Chart of RMC Plant.

3. Reliability Measures of Ready-Mix Concrete (RMC) Plant

3.1 Transition Probabilities

The state space of the RMC Plant is discrete, consisting of states $\{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9\}$. The transition from state 'i' to 'j' is represented by p_{ij}. By simple probabilistic factors, value of p_{ij} is found by following statements for the non-zero elements of RMC:

$$p_{ij}(t) = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

In the present system, the associated matric of transition probability is defined as:

$$X = \begin{bmatrix} p_{00} & \cdots & p_{08} \\ \vdots & \ddots & \vdots \\ p_{90} & \cdots & p_{99} \end{bmatrix}$$

So, equation (1) provides the values of all the statements of RMC (X) as:

$Q_{01}(t) = \int_0^\infty f_0(t)\overline{f_1(t)f_2(t)f_3(t)f_4(t)f_5(t)}dt$

Taking Laplace transform from both side

$$Q_{01}^{**}(s) = \int_0^\infty \Phi_0 \eta t^{\eta-1} e^{-(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5+s)t^\eta} dt$$

$$= \lim_{s\to 0} \frac{\Phi_0}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5+s)}$$

$\Rightarrow p_{01} = \lim_{s\to 0} Q_{01}^{**}(s) = \frac{\Phi_0}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$ $\quad p_{02} = \frac{\Phi_1}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$,

$p_{03} = \frac{\Phi_2}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$, $\quad p_{04} = \frac{\Phi_3}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$,

$p_{05} = \frac{\Phi_4}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$, $\quad p_{06} = \frac{\Phi_5}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)}$,

$p_{10} = \lim_{s\to 0} Q_{10}^{**}(s) = 1$, $\quad p_{20} = \lim_{s\to 0} Q_{20}^{**}(s) = 1$,

$p_{30} = \lim_{s\to 0} Q_{30}^{**}(s) = 1$, $\quad p_{40} = \lim_{s\to 0} Q_{40}^{**}(s) = 1$, $\quad p_{50} = \lim_{s\to 0} Q_{50}^{**}(s) = 1$,

$p_{67} = \frac{\Phi_6}{(\Phi_6+\Phi_7+\Phi_8)}$, $\quad p_{68} = \frac{\Phi_7}{(\Phi_6+\Phi_7+\Phi_8)}$, $\quad p_{69} = \frac{\Phi_8}{(\Phi_6+\Phi_7+\Phi_8)}$

$p_{70} = \lim_{s\to 0} Q_{70}^{**}(s) = 1$, $\quad p_{80} = \lim_{s\to 0} Q_{80}^{**}(s) = 1$, $\quad p_{90} = \lim_{s\to 0} Q_{90}^{**}(s) = 1$,

It is clearly confirmed that the sum of all statements of each row is unity.

3.2 Mean Sojourn Time

\` Mean sojourn time is the average time consumed by a system in a particular state. If T_i represent the average sojourn/survival time of RMC at a specific state S_i, then the mean sojourn time in the state S_i is assessed with following expressions:

$$\mu_i = \int_0^\infty P_r(T_i > t) = \sum_j m_{ij} \tag{2}$$

Where $m_{ij} = -\frac{d}{ds}\left[Q_{ij}^{**}(s)\right]_{s=0}$

Using equation (2), mean sojourn time μ_0 at state S_0 is calculated as follows:

$$\mu_0 = \int_0^\infty \overline{F_0(t)F_1(t)F_2(t)F_3(t)F_4(t)F_5(t)}dt \tag{3}$$

Taking Laplace transform on equation (3) both sides,

$$\mu_0^{**}(s) = \int_0^\infty e^{-\Phi_0 t^\eta} e^{-\Phi_1 t^\eta} e^{-\Phi_2 t^\eta} e^{-\Phi_3 t^\eta} e^{-\Phi_4 t^\eta} e^{-\Phi_5 t^\eta} e^{-st} dt$$

After solving it,

$$\mu_0^{**}(s) = \lim_{s\to 0} \frac{\Gamma(1+1/\eta)}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5+s)^{1/\eta}} \Rightarrow \mu_0 = \frac{\Gamma(1+1/\eta)}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)^{1/\eta}}$$

Similarly,

$$\mu_1 = \frac{\Gamma(1+1/\eta)}{(\psi_0)^{1/\eta}}, \qquad \mu_2 = \frac{\Gamma(1+1/\eta)}{(\psi_1)^{1/\eta}}, \qquad \mu_3 = \frac{\Gamma(1+1/\eta)}{(\psi_2)^{1/\eta}},$$

$$\mu_4 = \frac{\Gamma(1+1/\eta)}{(\psi_3)^{1/\eta}}, \qquad \mu_5 = \frac{\Gamma(1+1/\eta)}{(\psi_4)^{1/\eta}}, \qquad \mu_6 = \frac{\Gamma(1+1/\eta)}{(\Phi_6+\Phi_7+\Phi_8)^{1/\eta}},$$

$$\mu_7 = \frac{\Gamma(1+1/\eta)}{(\psi_5)^{1/\eta}}, \qquad \mu_8 = \frac{\Gamma(1+1/\eta)}{(\psi_6)^{1/\eta}}, \qquad \mu_9 = \frac{\Gamma(1+1/\eta)}{(\psi_7)^{1/\eta}}.$$

3.3 Mean Time to System Failure (MTSF)

To assess reliability $R_i(t)$ of RMC Plant at time "t" initiating from regenerative state S_i to a failed state S_j, it characterizes the cumulative density function of first passage time. The failed state is assumed as absorbing state, the following recursive relation for $R_i(t)$ are obtained applying probabilistic arguments:

$$R_0(t) = Z_0(t) \tag{4}$$

where,

$$Z_0(t) = \overline{Q_{01}(t)} * \overline{Q_{02}(t)} * \overline{Q_{03}(t)} * \overline{Q_{04}(t)} * \overline{Q_{05}(t)} \tag{5}$$

By using Laplace transform of equation (4)

$$R_0^{**}(s) = Z_0^{**}(s) \tag{6}$$

where,

$$Z_0^{**}(s) = \frac{\Gamma(1+1/n)}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)^{1/\eta}} \tag{7}$$

$$\text{MTSF} = \lim_{s\to 0} R_0^{**}(s) = \mu_0 = \frac{\Gamma(1+1/\eta)}{(\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5)^{1/\eta}} \tag{8}$$

3.4 Availability Analysis

The probability $A_i(t)$ of RMC plant, which refers that it is in up-state at time 't' provided that the system entered regenerative state S_i at t = 0. The recursive relations are as follows:

$$A_0(t) = Z_0(t) + Q_{01}(t)®A_1(t) + Q_{02}(t)®A_2(t) + Q_{03}(t)®A_3(t) + Q_{04}(t)®A_4(t) + Q_{05}(t)®A_5(t) + Q_{06}(t)®A_6(t) \tag{9}$$

$$A_1(t) = Q_{10}(t)®A_0(t) \tag{10}$$

$$A_2(t) = Q_{20}(t)®A_0(t) \tag{11}$$

$$A_3(t) = Q_{30}(t)®A_0(t) \tag{12}$$

$$A_4(t) = Q_{40}(t)®A_0(t) \tag{13}$$

$$A_5(t) = Q_{50}(t)®A_0(t) \tag{14}$$

$$A_6(t) = Q_{67}(t)®A_7(t) + Q_{68}(t)®A_8(t) + Q_{69}(t)®A_9(t) \tag{15}$$

$$A_7(t) = Q_{70}(t)®A_0(t) \tag{16}$$

$$A_8(t) = Q_{80}(t)®A_0(t) \tag{17}$$

$$A_9(t) = Q_{90}(t)®A_0(t) \tag{18}$$

Using Laplace transformation on equation (9-18),

$$A_0^{**}(s) = \frac{\mu_0}{\mu_0 + P_{01}\mu_1 + P_{02}\mu_2 + P_{03}\mu_3 + P_{04}\mu_4 + P_{05}\mu_5 + P_{06}[\mu_6 + P_{67}\mu_7 + P_{68}\mu_8 + P_{69}\mu_9]} \tag{19}$$

3.5 Busy Period of Server

The probability $B_i(t)$ that repairman is engaged in restoring the failed unit at epoch "t" provided that the RMC Plant entered in state S_i at t = 0. The recursive relations are given as:

$$B_0(t) = Q_{01}(t)®B_1(t) + Q_{02}(t)®B_2(t) + Q_{03}(t)®B_3(t) + Q_{04}(t)®B_4(t) + Q_{05}(t)®B_5(t) + Q_{06}(t)®B_6(t) \tag{20}$$

$$B_1(t) = Z_1(t) + Q_{10}(t)®B_0(t) \tag{21}$$

$$B_2(t) = Z_2(t) + Q_{20}(t)®B_0(t) \tag{22}$$

$$B_3(t) = Z_3(t) + Q_{30}(t)®B_0(t) \tag{23}$$

$$B_4(t) = Z_4(t) + Q_{40}(t)®B_0(t) \tag{24}$$

$$B_5(t) = Z_5(t) + Q_{50}(t)®B_0(t) \tag{25}$$

$$B_6(t) = Z_6(t) + Q_{67}(t)®B_7(t) + Q_{68}(t)®B_8(t) + Q_{69}(t)®B_9(t) \tag{26}$$

$$B_7(t) = Z_7(t) + Q_{70}(t)®B_0(t) \tag{27}$$

$$B_8(t) = Z_8(t) + Q_{80}(t)®B_0(t) \tag{28}$$

$$B_9(t) = Z_9(t) + Q_{90}(t)®B_0(t) \tag{29}$$

Using Laplace transformation on equations (20-29),

$$B_0^{**}(s) = \frac{N_2(s)}{D_1(s)}$$

where,

$$N_2(s) = Q_{01}^{**}(s)Z_1^{**}(s) + Q_{02}^{**}(s)Z_2^{**}(s) + Q_{03}^{**}(s)Z_3^{**}(s) + Q_{04}^{**}(s)Z_4^{**}(s) + Q_{05}^{**}(s)Z_5^{**}(s) + Q_{06}^{**}(s)[Z_6^{**}(s) + Q_{67}^{**}(s)Z_7^{**}(s) + Q_{68}^{**}(s)Z_8^{**}(s) + Q_{69}^{**}(s)Z_9^{**}(s)]$$

Busy period of server = $\lim_{s \to 0} B_0^{**}(s) = \lim_{s \to 0} \frac{N_2 + s\, N_2'}{D_1'} =$

$$= \frac{P_{01}\mu_1 + P_{02}\mu_2 + P_{03}\mu_3 + P_{04}\mu_4 + P_{05}\mu_5 + P_{06}[\mu_6 + P_{67}\mu_7 + P_{68}\mu_8 + P_{69}\mu_9]}{\mu_0 + P_{01}\mu_1 + P_{02}\mu_2 + P_{03}\mu_3 + P_{04}\mu_4 + P_{05}\mu_5 + P_{06}[\mu_6 + P_{67}\mu_7 + P_{68}\mu_8 + P_{69}\mu_9]} \quad (30)$$

3.6 Profit Function

The expected profit P earned by the system in extended life is

$$P = K_0 A_0 - K_1 B_0 \quad (31)$$

where K_0: income per unit time; K_1: cost per unit time

4. Reliability characteristics estimation under Classical and Bayesian framework

4.1 Estimation under Classical framework

Weibull distribution is assumed for the failure (f_i (.); i= 0,1, 2, 3, 4, 5, 6, 7, 8) and repair (g_i (.); i= 0, 1, 2, 3, 4, 5, 6, 7) rate of various components of RMC plant with different scale and common shape parameters.

$$f_i(t) = \Phi_i \eta t^{\eta-1}; i = 0,1,2,3,4,5,6,7,8$$

$$g_i(t) = \psi_i \eta t^{\eta-1}; i = 0,1,2,3,4,5,6,7$$

In this context, the common scale parameter is denoted as η while Φ_i & ψ_i represent scale parameters. All these random variables are assumed to be statistically independent. The primary objective of the current study is to estimate these parameters and assess the reliability measures of the RMC plant using both classical and Bayesian inferential methods. For the classical approach, the maximum likelihood (ML) estimation method is utilized due to its effectiveness in parameter estimation. The maximum likelihood estimators (MLEs) are derived for all the parameters associated with the random variables.

Seventeen independent random samples of size n_i (i = 1, 2, 3,……, 17) are taken from Weibull distribution with failure rates (f_i (.); i = 0, 1, 2, 3, 4, 5, 6, 7, 8) and repair (g_i (.); i = 0, 1, 2, 3, 4, 5, 6, 7) respectively.

$\widehat{Y_1} = (y_{11}, y_{12}, \dots \dots y_{1n_1})$ $\qquad$ $\widehat{Y_2} = (y_{21}, y_{22}, \dots \dots y_{2n_2})$

$\widehat{Y_3} = \left(y_{31}, y_{32}, \dots \dots y_{3n_3}\right)$ $\qquad$ $\widehat{Y_4} = \left(y_{41}, y_{42}, \dots \dots y_{4n_4}\right)$

$\widehat{Y_5} = (y_{51}, y_{52}, \dots \dots y_{5n_5})$ $\qquad$ $\widehat{Y_6} = (y_{61}, y_{62}, \dots \dots y_{6n_6})$

$\widehat{Y_7} = (y_{71}, y_{72}, \dots \dots y_{7n_7})$ $\qquad$ $\widehat{Y_8} = (y_{81}, y_{82}, \dots \dots y_{8n_8})$

$\widehat{Y_9} = \left(y_{91}, y_{92}, \dots \dots y_{9n_9}\right)$ $\qquad$ $\widehat{Y_{10}} = \left(y_{10,1}, y_{10,2}, \dots \dots y_{10,n_{10}}\right)$

$\widehat{Y_{11}} = (y_{11,1}, y_{11,2}, \dots \dots y_{11n_{11}})$ $\qquad$ $\widehat{Y_{12}} = (y_{12,1}, y_{12,2}, \dots \dots y_{12,n_{12}})$

$\widehat{Y_{13}} = (y_{13,1}, y_{13,2}, \dots \dots y_{13n_{13}})$ $\qquad$ $\widehat{Y_{14}} = (y_{14,1}, y_{14,2}, \dots \dots y_{14,n_{14}})$

$\widehat{Y_{15}} = (y_{15,1}, y_{15,2}, \dots \dots y_{15n_{15}})$ $\qquad$ $\widehat{Y_{16}} = (y_{16,1}, y_{16,2}, \dots \dots y_{16,n_{16}})$

$\widehat{Y_{17}} = (y_{17,1}, y_{17,2}, \dots \dots y_{17n_{17}})$

The likelihood function L is given by

$$L = L\left(\widehat{Y_1}, \widehat{Y_2}, \widehat{Y_3}, \dots \dots \dots, \widehat{Y_{16}}, \widehat{Y_{17}} \middle| \Phi_0, \Phi_1, \Phi_2, \dots, \Phi_8, \psi_0, \psi_1, \psi_2, \dots \dots, \psi_7\right)$$

$$L = {\Phi_0}^{n_1} {\Phi_1}^{n_2} \dots . {\Phi_8}^{n_9}\ {\psi_0}^{n_{10}} {\psi_1}^{n_{11}} \dots . {\psi_7}^{n_{17}}\ \eta^{n_1+n_2+\cdots+n_{16}+n_{17}} . S_1 . S_2 \dots . S_{16} . S_{17} . e^{-(\Phi_0 T_1 + \Phi_1 T_2 + \dots\dots + \Phi_8 T_9 + \psi_0 T_{10} + \psi_1 T_{11} + \dots . + \psi_7 T_{17})} \qquad (32)$$

Where

$$S_i = \prod_{j=1}^{n_i} y_{ij}^{\eta-1} \qquad and \qquad T_i = \sum_{j=1}^{n_i} y_{ij}^{\eta}, \qquad i = 1, 2, \dots \dots, 17$$

using log on both sides of the equation (32),

$$\log L = n_1 \log \Phi_0 + \cdots + n_{17} \log \psi_7 + \sum n_i \log \eta + \sum \log S_i - (\Phi_0 T_1 + \Phi_1 T_2 + \cdots + \psi_7 T_{17}) \qquad (33)$$

The ML estimates (say $\widehat{\Phi}_0, \widehat{\Phi}_1, \widehat{\Phi}_2, \widehat{\Phi}_3, \widehat{\Phi}_4, \widehat{\Phi}_5, \widehat{\Phi}_6, \widehat{\Phi}_7, \widehat{\Phi}_8, \widehat{\psi}_0, \widehat{\psi}_1, \widehat{\psi}_2, \widehat{\psi}_3, \widehat{\psi}_4, \widehat{\psi}_5, \widehat{\psi}_6, \widehat{\psi}_7$) of the shape and scale parameters $\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_8, \psi_0, \psi_1, \psi_2, \dots \dots, \psi_7$.

$$\widehat{\Phi}_0 = \frac{n_1}{\sum_{j=1}^{n_1} y_{ij}^{\eta}};\ \widehat{\Phi}_1 = \frac{n_2}{\sum_{j=1}^{n_2} y_{ij}^{\eta}};\ \widehat{\Phi}_2 = \frac{n_3}{\sum_{j=1}^{n_3} y_{ij}^{\eta}};\ \widehat{\Phi}_3 = \frac{n_4}{\sum_{j=1}^{n_4} y_{ij}^{\eta}};\ \widehat{\Phi}_4 = \frac{n_5}{\sum_{j=1}^{n_5} y_{ij}^{\eta}};\ \widehat{\Phi}_5 = \frac{n_6}{\sum_{j=1}^{n_6} y_{ij}^{\eta}};$$

$$\widehat{\Phi}_6 = \frac{n_7}{\sum_{j=1}^{n_7} y_{ij}^{\eta}};\ \widehat{\Phi}_7 = \frac{n_8}{\sum_{j=1}^{n_8} y_{ij}^{\eta}};\ \widehat{\Phi}_8 = \frac{n_9}{\sum_{j=1}^{n_9} y_{ij}^{\eta}};\ \widehat{\psi}_0 = \frac{n_{10}}{\sum_{j=1}^{n_{10}} y_{ij}^{\eta}};\ \widehat{\psi}_1 = \frac{n_{11}}{\sum_{j=1}^{n_{11}} y_{ij}^{\eta}};\ \widehat{\psi}_2 = \frac{n_{12}}{\sum_{j=1}^{n_{12}} y_{ij}^{\eta}};$$

$$\widehat{\psi}_3 = \frac{n_{13}}{\sum_{j=1}^{n_{13}} y_{ij}^{\eta}};\ \widehat{\psi}_4 = \frac{n_{14}}{\sum_{j=1}^{n_{14}} y_{ij}^{\eta}};\ \widehat{\psi}_5 = \frac{n_{15}}{\sum_{j=1}^{n_{15}} y_{ij}^{\eta}};\ \widehat{\psi}_6 = \frac{n_{16}}{\sum_{j=1}^{n_{16}} y_{ij}^{\eta}};\ \widehat{\psi}_7 = \frac{n_{17}}{\sum_{j=1}^{n_{17}} y_{ij}^{\eta}}; \qquad (34)$$

By employing the invariance property of MLE, the MLEs for functions of parameters can be easily obtained. This directly derives the expressions for MLEs of MTSF, availability and profit function. Here $\widehat{MTSF}$, $\widehat{AV}$ and $\hat{P}$ represented the MLE of MTSF, availability and profit function respectively. The asymptotic distribution is

$$\left(\widehat{\Phi}_0 - \Phi_0, \widehat{\Phi}_1 - \Phi_1, \widehat{\Phi}_2 - \Phi_2, \ldots \ldots \ldots .\widehat{\psi}_7 - \psi_7\right)' \sim N_{17}(0, I^{-1})$$

Here, I^{-1} represented the Fisher information matrix having diagonal elements,

$I_{11} = \frac{n_1}{\Phi_0^2},\ I_{22} = \frac{n_2}{\Phi_1^2},\ I_{33} = \frac{n_3}{\Phi_2^2},\ I_{44} = \frac{n_4}{\Phi_3^2},\ I_{55} = \frac{n_5}{\Phi_4^2},\quad I_{66} = \frac{n_6}{\Phi_5^2},\ I_{77} = \frac{n_7}{\Phi_6^2},\ I_{88} = \frac{n_8}{\Phi_7^2},\ I_{99} = \frac{n_9}{\Phi_8^2},$

$I_{10,10} = \frac{n_{10}}{\psi_0^2},\ I_{11,11} = \frac{n_{11}}{\psi_1^2},\ I_{12,12} = \frac{n_{12}}{\psi_2^2},\ I_{13,13} = \frac{n_{13}}{\psi_3^2},\ I_{14,14} = \frac{n_{14}}{\psi_4^2},$

$I_{15,15} = \frac{n_{15}}{\psi_5^2},\ I_{16,16} = \frac{n_{16}}{\psi_6^2},\ I_{17,17} = \frac{n_{17}}{\psi_7^2}.$

And the values of the remaining elements are zero.

The asymptotic distribution of MTSF, availability and profit are given by:

$$\left(\widehat{MTSF} - MTSF\right)' \sim N_{17}(0, A'I^{-1}A);\ \left(\widehat{AV} - AV\right)' \sim N_{17}(0, B'I^{-1}B);\ \left(\hat{P} - P\right)' \sim N_{17}(0, C'I^{-1}C)$$

where

$$A' = \begin{pmatrix} \frac{\partial MTSF}{\partial \Phi_0}, \frac{\partial MTSF}{\partial \Phi_1}, \frac{\partial MTSF}{\partial \Phi_2}, \frac{\partial MTSF}{\partial \Phi_3}, \frac{\partial MTSF}{\partial \Phi_4}, \frac{\partial MTSF}{\partial \Phi_5}, \frac{\partial MTSF}{\partial \Phi_6}, \frac{\partial MTSF}{\partial \Phi_7}, \frac{\partial MTSF}{\partial \Phi_8}, \\ \frac{\partial MTSF}{\partial \psi_0}, \frac{\partial MTSF}{\partial \psi_1}, \frac{\partial MTSF}{\partial \psi_2}, \frac{\partial MTSF}{\partial \psi_3}, \frac{\partial MTSF}{\partial \psi_4}, \frac{\partial MTSF}{\partial \psi_5}, \frac{\partial MTSF}{\partial \psi_6}, \frac{\partial MTSF}{\partial \psi_7} \end{pmatrix}'$$

$$B' = \begin{pmatrix} \frac{\partial AV}{\partial \Phi_0}, \frac{\partial AV}{\partial \Phi_1}, \frac{\partial AV}{\partial \Phi_2}, \frac{\partial AV}{\partial \Phi_3}, \frac{\partial AV}{\partial \Phi_4}, \frac{\partial AV}{\partial \Phi_5}, \frac{\partial AV}{\partial \Phi_6}, \frac{\partial AV}{\partial \Phi_7}, \frac{\partial AV}{\partial \Phi_8}, \\ \frac{\partial AV}{\partial \psi_0}, \frac{\partial AV}{\partial \psi_1}, \frac{\partial AV}{\partial \psi_2}, \frac{\partial AV}{\partial \psi_3}, \frac{\partial AV}{\partial \psi_4}, \frac{\partial AV}{\partial \psi_5}, \frac{\partial AV}{\partial \psi_6}, \frac{\partial AV}{\partial \psi_7} \end{pmatrix}'$$

$$C' = \begin{pmatrix} \frac{\partial P}{\partial \Phi_0}, \frac{\partial P}{\partial \Phi_1}, \frac{\partial P}{\partial \Phi_2}, \frac{\partial P}{\partial \Phi_3}, \frac{\partial P}{\partial \Phi_4}, \frac{\partial P}{\partial \Phi_5}, \frac{\partial P}{\partial \Phi_6}, \frac{\partial P}{\partial \Phi_7}, \frac{\partial P}{\partial \Phi_8}, \\ \frac{\partial P}{\partial \psi_0}, \frac{\partial P}{\partial \psi_1}, \frac{\partial P}{\partial \psi_2}, \frac{\partial P}{\partial \psi_3}, \frac{\partial P}{\partial \psi_4}, \frac{\partial P}{\partial \psi_5}, \frac{\partial P}{\partial \psi_6}, \frac{\partial P}{\partial \psi_7} \end{pmatrix}'$$

4.2 Estimation under Bayesian framework

In Bayesian estimation of the parameters and reliability measures of the RMC plant, it is assumed that all parameters associated with failure and repair rates follow specific probability distributions. In this study, all the random variables are modeled using a two-parameter Weibull distribution with a known shape parameter η. The scale parameters $(\Phi_0, \Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7, \Phi_8, \psi_0, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7)$ associated with these random variables are assumed to follow a Gamma distribution characterized by hyperparameters (α_i, β_i; i = 1, 2,……17) as described below:

$\Phi_0 \sim GAMMA\ (\alpha_1, \beta_1)\qquad \Phi_1 \sim GAMMA\ (\alpha_2, \beta_2)\qquad \Phi_2 \sim GAMMA\ (\alpha_3, \beta_3)$

$\Phi_3 \sim GAMMA\ (\alpha_4, \beta_4)$ $\Phi_4 \sim GAMMA\ (\alpha_5, \beta_5)$ $\Phi_5 \sim GAMMA\ (\alpha_6, \beta_6)$

$\Phi_6 \sim GAMMA\ (\alpha_7, \beta_7)$ $\Phi_7 \sim GAMMA\ (\alpha_8, \beta_8)$ $\Phi_8 \sim GAMMA\ (\alpha_9, \beta_9)$

$\psi_0 \sim GAMMA\ (\alpha_{10}, \beta_{10})$ $\psi_1 \sim GAMMA\ (\alpha_{11}, \beta_{11})$ $\psi_2 \sim GAMMA\ (\alpha_{12}, \beta_{12})$

$\psi_3 \sim GAMMA\ (\alpha_{13}, \beta_{13})$ $\psi_4 \sim GAMMA\ (\alpha_{14}, \beta_{14})$ $\psi_5 \sim GAMMA\ (\alpha_{15}, \beta_{15})$

$\psi_6 \sim GAMMA\ (\alpha_{16}, \beta_{16})$ $\psi_7 \sim GAMMA\ (\alpha_{17}, \beta_{17})$

By likelihood function (32), the posterior distributions are derived and the prior distributions of $(\Phi_0, \Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7, \Phi_8, \psi_0, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7)$ are as follows:

$$\Phi_0|\underline{Y_1} \sim GAMMA(n_1 + \alpha_1, \beta_1 + \textstyle\sum_{j=1}^{n_1} y_{ij}^{\eta}) \tag{35}$$

$$\Phi_1|\underline{Y_2} \sim GAMMA(n_2 + \alpha_2, \beta_2 + \textstyle\sum_{j=1}^{n_2} y_{ij}^{\eta}) \tag{36}$$

$$\Phi_2|\underline{Y_3} \sim GAMMA(n_3 + \alpha_3, \beta_3 + \textstyle\sum_{j=1}^{n_3} y_{ij}^{\eta}) \tag{37}$$

$$\Phi_3|\underline{Y_4} \sim GAMMA(n_4 + \alpha_4, \beta_4 + \textstyle\sum_{j=1}^{n_4} y_{ij}^{\eta}) \tag{38}$$

$$\Phi_4|\underline{Y_5} \sim GAMMA(n_5 + \alpha_5, \beta_5 + \textstyle\sum_{j=1}^{n_5} y_{ij}^{\eta}) \tag{39}$$

$$\Phi_5|\underline{Y_6} \sim GAMMA(n_6 + \alpha_6, \beta_6 + \textstyle\sum_{j=1}^{n_6} y_{ij}^{\eta}) \tag{40}$$

$$\Phi_6|\underline{Y_7} \sim GAMMA(n_7 + \alpha_7, \beta_7 + \textstyle\sum_{j=1}^{n_7} y_{ij}^{\eta}) \tag{41}$$

$$\Phi_7|\underline{Y_8} \sim GAMMA(n_8 + \alpha_8, \beta_8 + \textstyle\sum_{j=1}^{n_8} y_{ij}^{\eta}) \tag{42}$$

$$\Phi_8|\underline{Y_9} \sim GAMMA(n_9 + \alpha_9, \beta_9 + \textstyle\sum_{j=1}^{n_9} y_{ij}^{\eta}) \tag{43}$$

$$\psi_0|\underline{Y_{10}} \sim GAMMA(n_{10} + \alpha_{10}, \beta_{10} + \textstyle\sum_{j=1}^{n_{10}} y_{ij}^{\eta}) \tag{44}$$

$$\psi_1|\underline{Y_{11}} \sim GAMMA(n_{11} + \alpha_{11}, \beta_{11} + \textstyle\sum_{j=1}^{n_{11}} y_{ij}^{\eta}) \tag{45}$$

$$\psi_2|\underline{Y_{12}} \sim GAMMA(n_{12} + \alpha_{12}, \beta_{12} + \textstyle\sum_{j=1}^{n_{12}} y_{ij}^{\eta}) \tag{46}$$

$$\psi_3|\underline{Y_{13}} \sim GAMMA(n_{13} + \alpha_{13}, \beta_{13} + \textstyle\sum_{j=1}^{n_{13}} y_{ij}^{\eta}) \tag{47}$$

$$\psi_4|\underline{Y_{14}} \sim GAMMA(n_{14} + \alpha_{14}, \beta_{14} + \textstyle\sum_{j=1}^{n_{14}} y_{ij}^{\eta}) \tag{48}$$

$$\psi_5|\underline{Y_{15}} \sim GAMMA(n_{15} + \alpha_{15}, \beta_{15} + \textstyle\sum_{j=1}^{n_{15}} y_{ij}^{\eta}) \tag{49}$$

$$\psi_6|\underline{Y_{16}} \sim GAMMA(n_{16} + \alpha_{16}, \beta_{16} + \textstyle\sum_{j=1}^{n_{16}} y_{ij}^{\eta}) \tag{50}$$

$$\psi_7|\underline{Y_{17}} \sim GAMMA(n_{17} + \alpha_{17}, \beta_{17} + \textstyle\sum_{j=1}^{n_{17}} y_{ij}^{\eta}) \tag{51}$$

The scale parameters Φ_0, Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, Φ_6, Φ_7, Φ_8, ψ_0, ψ_1, ψ_2, ψ_3, ψ_4, ψ_5, ψ_6, ψ_7 and the Bayesian estimator of these parameter, equations (35)-(51) provide the means of the posterior distribution within squared error loss function which are as follows:

$$\hat{\Phi}_0 = \frac{\beta_1 + \sum_{j=1}^{n_1} y_{ij}^{\eta}}{n_1 + \alpha_1}; \quad \hat{\Phi}_1 = \frac{\beta_2 + \sum_{j=1}^{n_2} y_{ij}^{\eta}}{n_2 + \alpha_2}; \quad \hat{\Phi}_2 = \frac{\beta_3 + \sum_{j=1}^{n_3} y_{ij}^{\eta}}{n_3 + \alpha_3}; \quad \hat{\Phi}_3 = \frac{\beta_4 + \sum_{j=1}^{n_4} y_{ij}^{\eta}}{n_4 + \alpha_4};$$

$$\hat{\Phi}_4 = \frac{\beta_5 + \sum_{j=1}^{n_5} y_{ij}^{\eta}}{n_5 + \alpha_5}; \quad \hat{\Phi}_5 = \frac{\beta_6 + \sum_{j=1}^{n_6} y_{ij}^{\eta}}{n_6 + \alpha_6}; \quad \hat{\Phi}_6 = \frac{\beta_7 + \sum_{j=1}^{n_7} y_{ij}^{\eta}}{n_7 + \alpha_7}; \quad \hat{\Phi}_7 = \frac{\beta_8 + \sum_{j=1}^{n_8} y_{ij}^{\eta}}{n_8 + \alpha_8};$$

$$\hat{\Phi}_8 = \frac{\beta_9 + \sum_{j=1}^{n_9} y_{ij}^{\eta}}{n_9 + \alpha_9}; \quad \hat{\psi}_0 = \frac{\beta_{10} + \sum_{j=1}^{n_{10}} y_{ij}^{\eta}}{n_{10} + \alpha_{10}}; \quad \hat{\psi}_1 = \frac{\beta_{11} + \sum_{j=1}^{n_{11}} y_{ij}^{\eta}}{n_{11} + \alpha_{11}}; \quad \hat{\psi}_2 = \frac{\beta_{12} + \sum_{j=1}^{n_{12}} y_{ij}^{\eta}}{n_{12} + \alpha_{12}};$$

$$\hat{\psi}_3 = \frac{\beta_{13} + \sum_{j=1}^{n_{13}} y_{ij}^{\eta}}{n_{13} + \alpha_{13}}; \quad \hat{\psi}_4 = \frac{\beta_{14} + \sum_{j=1}^{n_{14}} y_{ij}^{\eta}}{n_{14} + \alpha_{14}}; \quad \hat{\psi}_5 = \frac{\beta_{15} + \sum_{j=1}^{n_{15}} y_{ij}^{\eta}}{n_{15} + \alpha_{15}}; \quad \hat{\psi}_6 = \frac{\beta_{16} + \sum_{j=1}^{n_{16}} y_{ij}^{\eta}}{n_{16} + \alpha_{16}};$$

$$\hat{\psi}_7 = \frac{\beta_{17} + \sum_{j=1}^{n_{17}} y_{ij}^{\eta}}{n_{17} + \alpha_{17}}$$

5. Simulation Study

In this section, the MLE and Bayesian estimates for parameters of Weibull distribution related to the failure and repair rates of RMC plant are presented. Specifically, the MLE and Bayes estimates of scale parameters Φ_0, Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, Φ_6, Φ_7, Φ_8, ψ_0, ψ_1, ψ_2, ψ_3, ψ_4, ψ_5, ψ_6, ψ_7 are derived. It is assumed that the scale parameter is known, the estimates for availability, MTSF and profit function are calculated by the invariance property. A simulation study is carried out to validate the theoretical results. The mean square error of estimates and the width of confidence intervals are compared. The investigation is also carried out for various values of the shape parameters since the Weibull distribution's hazard rate is rising, falling and constant, depending on the shape parameter. The Weibull distribution is applied to generate a random sample of size 50. The samples are generated through simulation for the set of values given below:

For η = 0.50, 1, 2 and n=50

- Φ_0= 0.007, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5= 0.0033, Φ_6= 0.003, Φ_7= 0.004, Φ_8= 0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5= 0.72, ψ_6= 0.59, ψ_7= 0.387
- Φ_0= 0.008, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5= 0.0033, Φ_6= 0.003, Φ_7= 0.004, Φ_8=0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.009, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5= 0.0033, Φ_6= 0.003, Φ_7 = 0.004, Φ_8=0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5= 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.010, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5= 0.0033, Φ_6= 0.003, Φ_7= 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3 = 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.011, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3 = 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.012, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3 = 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.013, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.014, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56,

ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387

- Φ_0= 0.015, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.016, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.017, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.018, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6 = 0.59, ψ_7= 0.387
- Φ_0= 0.019, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5= 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8= 0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5= 0.72, ψ_6= 0.59, ψ_7= 0.387
- Φ_0= 0.020, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6= 0.59, ψ_7= 0.387
- Φ_0= 0.021, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6= 0.59, ψ_7= 0.387
- Φ_0= 0.022, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6= 0.59, ψ_7= 0.387
- Φ_0= 0.023, Φ_1= 0.005, Φ_2= 0.0045, Φ_3= 0.0062, Φ_4= 0.0039, Φ_5 = 0.0033, Φ_6 = 0.003, Φ_7 = 0.004, Φ_8 =0.006, ψ_0= 0.02, ψ_1= 0.4, ψ_2= 0.56, ψ_3= 0.432, ψ_4= 0.39, ψ_5 = 0.72, ψ_6= 0.59, ψ_7= 0.387

Using the parameter values mentioned above, thirty random samples are generated, and MLE as well as Bayesian estimation for non-informative prior are performed for the parameters, availability, MTSF and profit function. For the Bayesian analysis, 10,000 realizations are generated by non-informative prior and posterior densities. The set of values for Gamma hyper parameters are obtained by establishing $\frac{\alpha}{\beta_i} = \frac{a_i}{b_i}$. All estimates along with true values, mean square errors, and length of intervals/ HPD, are summarized in table 1-9. The profit function is evaluated using k_1 and k_2 as 10000 and 1200, respectively. All numerical computations are performed using programs developed in R-environment.

Table 1. Value of MTSF for fixed eta =0.5 and varying ϕ0

Estimates ϕ0	mtsf	mtsf.mse. mle	mtsf.AE. mle	mtsf.AE. bayesf	mtsf.mse. bayesf	mtsf.length. mle	mtsf.length. bayes
0.007	2237.111	70899.924	2168.653	2003.374	109705.836	1040.479	1043.255
0.008	2094.658	70714.440	2036.183	1885.156	99832.651	986.375	992.734
0.009	1965.390	56663.044	1916.738	1779.285	79862.512	940.158	948.886
0.010	1847.729	53392.575	1784.423	1661.635	75763.142	890.028	900.759
0.011	1740.326	48800.129	1699.062	1586.163	62890.204	859.474	872.391
0.012	1642.023	43581.333	1595.180	1493.804	56369.995	820.939	835.341
0.013	1551.819	44501.873	1502.381	1411.541	54734.012	787.784	803.391
0.014	1468.849	39958.687	1435.962	1352.704	45754.809	763.362	780.988
0.015	1392.360	39708.648	1347.211	1273.918	45233.837	731.102	751.133
0.016	1321.694	33649.652	1288.588	1221.973	36905.619	709.881	730.548
0.017	1256.274	31645.277	1232.188	1172.426	32787.450	690.888	713.192
0.018	1195.593	27309.932	1169.353	1116.168	28464.585	665.218	688.280
0.019	1139.205	27155.997	1108.648	1062.271	27738.542	641.430	666.340
0.020	1086.714	26719.720	1059.209	1018.719	26193.024	623.317	648.966
0.021	1037.770	22268.552	1018.998	982.800	21206.759	607.060	634.189
0.022	992.059	23033.010	963.689	932.868	21962.687	582.166	609.649
0.023	949.303	21230.151	925.898	899.427	19723.047	567.419	597.107

Table 2. Values of Availability fo r fixed eta =0.5 and varying ϕ0

Estimates ϕ0	Av	Av.mse. mle	Av.AE. mle	Av.AE. bayes	Av.mse. bayes	Av.length. mle	Av.length. bayes
0.007	0.6279	0.0029	0.6270	0.6221	0.0027	0.2189	0.2294
0.008	0.6199	0.0030	0.6180	0.6132	0.0028	0.2206	0.2313
0.009	0.6120	0.0031	0.6115	0.6067	0.0029	0.2217	0.2319
0.010	0.6044	0.0034	0.6026	0.5977	0.0032	0.2229	0.2331
0.011	0.5969	0.0031	0.5975	0.5928	0.0029	0.2239	0.2339
0.012	0.5896	0.0033	0.5923	0.5875	0.0030	0.2245	0.2346
0.013	0.5824	0.0033	0.5822	0.5776	0.0031	0.2262	0.2360
0.014	0.5754	0.0032	0.5716	0.5673	0.0030	0.2278	0.2373
0.015	0.5686	0.0033	0.5675	0.5631	0.0030	0.2281	0.2373
0.016	0.5619	0.0031	0.5617	0.5574	0.0028	0.2289	0.2382
0.017	0.5553	0.0033	0.5558	0.5517	0.0030	0.2294	0.2385
0.018	0.5489	0.0033	0.5471	0.5433	0.0030	0.2303	0.2395
0.019	0.5426	0.0035	0.5426	0.5388	0.0031	0.2304	0.2392
0.020	0.5364	0.0033	0.5357	0.5321	0.0030	0.2312	0.2398
0.021	0.5304	0.0034	0.5325	0.5290	0.0031	0.2312	0.2399
0.022	0.5244	0.0033	0.5238	0.5205	0.0029	0.2318	0.2405
0.023	0.5186	0.0034	0.5206	0.5174	0.0030	0.2320	0.2404

Table 3. Values of Profit for fixed eta =0.5 and varying ϕ0

Estimates ϕ0	Pr	Pr.mse. mle	Pr.AE. mle	Pr.AE. bayes	Pr.mse. bayes	Pr.length. mle	Pr.length. bayes
0.007	5777.572	359824.043	5768.229	5713.514	337470.080	2448.681	2566.083
0.008	5689.158	374111.110	5669.035	5614.519	351581.218	2467.599	2585.849
0.009	5602.745	388454.231	5597.428	5543.041	361806.758	2478.999	2592.889
0.010	5518.269	423159.514	5498.699	5444.307	394152.594	2492.078	2605.261
0.011	5435.666	387730.683	5442.743	5389.266	358327.156	2502.754	2613.440
0.012	5354.876	414918.505	5385.889	5331.714	379699.409	2508.649	2621.588
0.013	5275.839	417905.577	5274.219	5222.618	384334.582	2527.065	2636.030
0.014	5198.502	402826.163	5156.152	5107.817	372691.467	2544.284	2649.591
0.015	5122.809	411850.883	5110.932	5061.383	378170.270	2547.619	2649.817
0.016	5048.711	385194.274	5047.055	4999.216	351294.099	2556.565	2659.611
0.017	4976.157	408107.903	4981.701	4935.401	369988.304	2560.893	2661.787
0.018	4905.102	407621.316	4885.374	4841.907	369745.228	2571.190	2672.710
0.019	4835.499	431841.948	4836.469	4793.294	390456.791	2571.635	2668.981
0.020	4767.305	416397.041	4759.531	4719.344	376640.622	2580.264	2675.866
0.021	4700.479	427143.240	4724.938	4684.785	381162.860	2580.536	2676.538
0.022	4634.981	407643.394	4627.535	4590.371	366437.225	2586.327	2682.169
0.023	4570.771	417464.212	4593.359	4556.097	371119.230	2588.232	2681.452

Table 4. Values of MTSF for fixed eta =1.0 and varying ϕ0

Estimates ϕ0	mtsf	mtsf.mse. mle	mtsf.AE. mle	mtsf.AE. bayes	mtsf.mse. bayes	mtsf.length. mle	mtsf.length. bayes
0.007	33.445	4.191	32.936	31.600	6.940	7.898	8.235
0.008	32.363	4.520	31.808	30.556	7.045	7.709	8.058
0.009	31.348	4.006	30.816	29.643	6.238	7.572	7.925
0.010	30.395	3.668	29.963	28.857	5.478	7.466	7.820
0.011	29.499	3.621	29.013	27.982	5.310	7.344	7.711
0.012	28.653	3.443	28.243	27.275	4.801	7.267	7.634
0.013	27.855	3.852	27.418	26.526	4.988	7.197	7.577
0.014	27.100	3.443	26.753	25.914	4.322	7.124	7.506
0.015	26.385	3.250	25.993	25.218	4.074	7.041	7.425
0.016	25.707	3.218	25.251	24.544	3.980	6.981	7.373
0.017	25.063	3.308	24.770	24.106	3.718	6.940	7.338
0.018	24.450	3.053	24.165	23.554	3.394	6.877	7.278
0.019	23.866	3.039	23.509	22.958	3.349	6.808	7.218
0.020	23.310	2.967	22.983	22.479	3.155	6.749	7.164
0.021	22.779	2.995	22.468	22.017	3.067	6.713	7.136
0.022	22.272	2.837	21.958	21.547	2.865	6.634	7.062
0.023	21.787	2.802	21.555	21.182	2.710	6.591	7.020

Table 5. Values of Availability for fixed eta =1.0 and varying ϕ0

Estimates ϕ0	Av	Av.mse. mle	Av.AE. mle	Av.AE. bayes	Av.mse. bayes	Av.length. mle	Av.length. bayes
0.007	0.7460	0.0006	0.7461	0.7135	0.0016	0.0967	0.0995
0.008	0.7432	0.0006	0.7430	0.7096	0.0016	0.0965	0.0995
0.009	0.7405	0.0005	0.7390	0.7051	0.0017	0.0966	0.0999
0.010	0.7378	0.0006	0.7371	0.7025	0.0017	0.0962	0.0996
0.011	0.7351	0.0006	0.7333	0.6980	0.0019	0.0963	0.1000
0.012	0.7324	0.0006	0.7309	0.6950	0.0019	0.0962	0.1000
0.013	0.7297	0.0006	0.7276	0.6912	0.0020	0.0962	0.1003
0.014	0.7270	0.0005	0.7260	0.6890	0.0019	0.0959	0.1003
0.015	0.7244	0.0006	0.7232	0.6856	0.0020	0.0959	0.1006
0.016	0.7218	0.0006	0.7197	0.6816	0.0021	0.0961	0.1012
0.017	0.7192	0.0006	0.7192	0.6805	0.0020	0.0956	0.1011
0.018	0.7166	0.0006	0.7161	0.6770	0.0021	0.0958	0.1015
0.019	0.7141	0.0006	0.7129	0.6733	0.0022	0.0960	0.1021
0.020	0.7115	0.0006	0.7101	0.6702	0.0022	0.0964	0.1028
0.021	0.7090	0.0006	0.7058	0.6656	0.0024	0.0970	0.1037
0.022	0.7065	0.0006	0.7045	0.6638	0.0023	0.0969	0.1039
0.023	0.7040	0.0006	0.7030	0.6618	0.0023	0.0969	0.1043

Table 6. Values of Profit for fixed eta =1.0 and varying ϕ0

Estimates ϕ0	Pr	Pr.mse. mle	Pr.AE. mle	Pr.AE. bayes	Pr.mse. bayes	Pr.length. mle	Pr.length. bayes
0.007	7133.169	74964.439	7133.766	6767.225	198110.751	1084.148	1114.206
0.008	7102.673	73450.315	7099.578	6724.033	206065.830	1081.565	1114.297
0.009	7072.375	68762.376	7055.788	6673.958	217258.826	1083.232	1118.223
0.01	7042.275	69590.086	7034.565	6645.207	217136.140	1078.406	1114.876
0.011	7012.375	74476.186	6992.246	6595.820	236323.857	1079.624	1118.978
0.012	6982.672	72385.362	6965.900	6562.882	237678.129	1077.864	1118.844
0.013	6953.168	73715.027	6930.261	6520.730	249872.568	1077.994	1122.645
0.014	6923.860	67752.105	6912.658	6496.469	240496.048	1074.219	1122.499
0.015	6894.750	72480.081	6881.561	6458.436	251960.362	1073.832	1125.472
0.016	6865.834	77607.554	6842.022	6413.404	270034.701	1075.760	1131.738
0.017	6837.113	71175.105	6837.180	6402.051	250371.191	1070.723	1130.495
0.018	6808.585	72784.242	6802.398	6362.735	260621.582	1073.167	1135.394
0.019	6780.250	73537.581	6767.087	6321.877	272411.012	1075.259	1142.105
0.02	6752.105	73183.820	6736.703	6287.134	277869.020	1079.057	1149.554
0.021	6724.150	70791.265	6688.133	6236.934	296931.279	1085.532	1159.053
0.022	6696.383	74227.357	6674.194	6215.950	292681.428	1085.146	1161.964
0.023	6668.803	76882.131	6657.081	6194.675	289695.268	1084.707	1166.084

Table 7. Values of MTSF for fixed eta =2.0 and varying ϕ0

Estimates ϕ0	mtsf	mtsf.mse. mle	mtsf.AE. mle	mtsf.AE. bayes	mtsf.mse. bayes	mtsf.length. mle	mtsf.length. bayes
0.007	5.125	0.025	5.082	4.975	0.044	0.609	0.648
0.008	5.042	0.026	5.009	4.908	0.041	0.607	0.647
0.009	4.962	0.024	4.919	4.822	0.041	0.604	0.644
0.01	4.886	0.027	4.843	4.751	0.041	0.604	0.645
0.011	4.813	0.025	4.771	4.684	0.038	0.604	0.645
0.012	4.744	0.026	4.697	4.615	0.038	0.606	0.648
0.013	4.677	0.026	4.638	4.560	0.036	0.609	0.651
0.014	4.614	0.025	4.575	4.501	0.034	0.610	0.653
0.015	4.552	0.029	4.508	4.438	0.037	0.611	0.655
0.016	4.493	0.027	4.453	4.388	0.034	0.616	0.660
0.017	4.437	0.028	4.393	4.331	0.034	0.617	0.660
0.018	4.382	0.030	4.343	4.286	0.034	0.621	0.665
0.019	4.330	0.029	4.293	4.240	0.032	0.622	0.667
0.02	4.279	0.029	4.249	4.200	0.031	0.625	0.670
0.021	4.230	0.027	4.199	4.154	0.028	0.626	0.672
0.022	4.182	0.026	4.155	4.114	0.027	0.629	0.675
0.023	4.137	0.028	4.102	4.064	0.028	0.630	0.677

Table 8. Values of Availability for fixed eta=2.o varying ϕ0

Estimates ϕ0	Av	Av.mse. mle	Av.AE. mle	Av.AE. bayes	Av.mse .bayes	Av.length. mle	Av.length. bayes
0.007	0.6868	0.0002	0.6861	0.6451	0.0019	0.0513	0.0530
0.008	0.6843	0.0002	0.6837	0.6427	0.0019	0.0509	0.0528
0.009	0.6819	0.0002	0.6814	0.6403	0.0019	0.0505	0.0525
0.010	0.6795	0.0002	0.6787	0.6376	0.0019	0.0504	0.0526
0.011	0.6771	0.0002	0.6757	0.6347	0.0019	0.0504	0.0528
0.012	0.6748	0.0002	0.6734	0.6324	0.0019	0.0503	0.0528
0.013	0.6725	0.0002	0.6713	0.6304	0.0019	0.0502	0.0530
0.014	0.6702	0.0002	0.6691	0.6281	0.0019	0.0503	0.0533
0.015	0.6680	0.0002	0.6668	0.6258	0.0019	0.0504	0.0534
0.016	0.6658	0.0002	0.6648	0.6239	0.0019	0.0505	0.0538
0.017	0.6637	0.0002	0.6626	0.6217	0.0019	0.0506	0.0540
0.018	0.6616	0.0002	0.6607	0.6199	0.0019	0.0509	0.0544
0.019	0.6595	0.0002	0.6579	0.6173	0.0019	0.0512	0.0549
0.020	0.6574	0.0002	0.6565	0.6159	0.0018	0.0514	0.0553
0.021	0.6554	0.0002	0.6549	0.6143	0.0018	0.0516	0.0556
0.022	0.6534	0.0002	0.6518	0.6115	0.0019	0.0522	0.0563
0.023	0.6514	0.0002	0.6505	0.6101	0.0018	0.0524	0.0566

Table 9. Values of Profit for fixed eta=2.0 and varying ϕ0

Estimates ϕ0	Pr	Pr.mse. mle	Pr.AE. mle	Pr.AE. bayes	Pr.mse. bayes	Pr.length. mle	Pr.length. bayes
0.007	6479.541	19790.365	6471.648	6011.526	234690.330	573.798	592.378
0.008	6452.085	19707.931	6445.053	5985.621	233343.839	569.389	589.452
0.009	6425.067	20795.494	6419.606	5958.289	234201.300	564.865	586.993
0.010	6398.477	21440.410	6390.041	5928.723	237484.833	563.827	588.107
0.011	6372.306	19417.868	6357.036	5896.779	241244.049	563.432	589.551
0.012	6346.543	20488.814	6331.406	5870.901	241956.465	561.993	590.064
0.013	6321.178	19521.635	6308.135	5848.839	237977.836	561.280	591.935
0.014	6296.202	18975.664	6283.258	5823.667	237970.115	562.389	594.959
0.015	6271.606	20938.909	6257.737	5798.034	240550.215	563.057	596.901
0.016	6247.378	20520.801	6236.037	5777.237	236807.794	564.442	600.864
0.017	6223.510	20652.543	6210.813	5752.696	237630.854	565.176	603.115
0.018	6199.993	20052.188	6190.183	5732.294	234257.067	568.745	608.038
0.019	6176.818	20764.168	6159.471	5703.459	239870.678	572.640	613.620
0.020	6153.977	20383.615	6143.996	5688.192	232393.148	575.057	618.058
0.021	6131.459	21557.820	6126.287	5670.334	228848.834	576.863	621.055
0.022	6109.258	21219.134	6091.615	5639.353	236413.408	583.471	628.577
0.023	6087.366	22110.020	6077.267	5623.427	231773.992	585.630	632.434

6. Conclusion

The current study employs both classical and Bayesian estimation strategies to assess the reliability characteristics of the RMC plant. The profit function, steady-state availability, and true MTSF are evaluated for a given set of parameter values. The findings, which are summed up in Tables 1–9, demonstrate that when the scale parameter of the maximum operation time Φ_0 rises, MTSF, availability, and profit all are decline. From the simulation results, it is observed that for the shape parameter, the true values of MTSF, availability, and profit, along with the MLE and Bayesian estimates of MTSF, availability, and profit, all decrease as the failure rate Φ_0 of the rolling belt unit increases. In comparison to the Bayesian MSE and HPD intervals for η=0.50, 1, and 2, the maximum likelihood estimators' mean square error (MSE) and the breadth of their confidence intervals for MTSF, availability, and profit are less. As a result, for the reliability characteristics of the RMC plant, Maximum Likelihood estimate (MLE) is advised above Bayesian estimate.

Conflicts of Interest

The authors declare no conflict of interest.

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