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ARTICLE

Two Echelon Supply Chain Model with lead time and effect of carbon emission under fuzzy environment[1]

Shivraj Singh[1], Anjali Gaur[2], Dhavendra Singh[3] and Dipti Singh[1]*.

[1]Department of Mathematics, Chaudhary Charan Singh University, Meerut, Uttar Pradesh, India.
[2]Hariom Saraswati PG College, Dhananauri, Haridwar, India.
[3]Department of Mechanical Engineering, Chaudhary Charan Singh University, Meerut, Uttar Pradesh, India.
*Corresponding author. Email: singhdipti113@gmail.com

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Abstract
Carbon emission has become a major challenge everywhere, such as in the production process, holding items, deteriorating items, waste disposal, and transporting items. Two warehouses are a realistic approach in inventory modeling. Inflation and lead time play a key role in making this study closer to reality. Uncertainty is also a very realistic approach for any organization. In this study, we developed a supply chain model in which there is one retailer and one supplier. The retailer holds its inventory in two warehouses. Our objective is to find the optimal total cost, cycle length, and supplier's lead time. In this study, we calculate the total cost in three different ways: first, for the crisp model; second, for the fuzzy model using the signed distance method; and third, using the graded mean integration method. We carried out the numerical solution using the software MATHEMATICA 12.0. From numerical illustration, we find that the total cost is minimum for the fuzzy model using the graded mean integration method. Sensitivity analysis is carried out to see the behavior of different parameters on the total cost.
Keywords: Carbon Emission, Lead Time, Two Warehouse, Inflation, Triangular Fuzzy Number.

1. Introduction & Literature Review

In today's business scenario, uncertainty is everywhere. In inventory modeling, uncertainty is present in holding costs, ordering costs, deterioration costs, shortage costs, etc. Due to uncertainty, the fuzzy concept is used in the proposed model. Deterioration is defined as the decay or deterioration of an object. Ghare and Schrader (1963) were the first to study the effect of deterioration. Ghare and Schrader's work was extended by Covert and Philip (1973). They introduced the concept of time-dependent deterioration. Park (1987) proposed the first fuzzy economic order quantity model. They assessed the impact of several techniques on obtaining the EOQ model. Yao and Lee (1999) discovered that the total cost after defuzzification (DFZ) is slightly greater than in the crisp model. In inventory modeling, some researchers have employed fuzzy ideas. Dutta et al. (2005), Jaggi et al. (2013) worked in the field of fuzzy environment. Sonia et al. (2016) and Priyan and Manivannan (2017), among others, have contributed to this field. Shee and Tripti (2020) formulated a fuzzy two-echelon supply chain model. They found in this study that when the signed distance method of defuzzification is utilized, the total cost and cycle duration are minimal. They also used the concept of variable holding cost.

Nayak et al. (2020) developed a fuzzy inventory model that also considers permissible delay in payment. Kumar and Paikray (2022) formulated a cost optimization inventory model that considers deteriorating items under a fuzzy environment. Carbon emission is becoming a major problem all over the world. The temperature of the earth is increasing day by day due to carbon emissions, which harms the environment. In today's business scenario, uncertainty is everywhere. In inventory modeling, uncertainty is present in holding costs, ordering costs, deterioration costs, shortage costs, etc. Due to uncertainty, the fuzzy concept is used in the proposed model. Deterioration is defined as the decay or deterioration of an object. Ghare and Schrader (1963) were the first to study the effect of deterioration.

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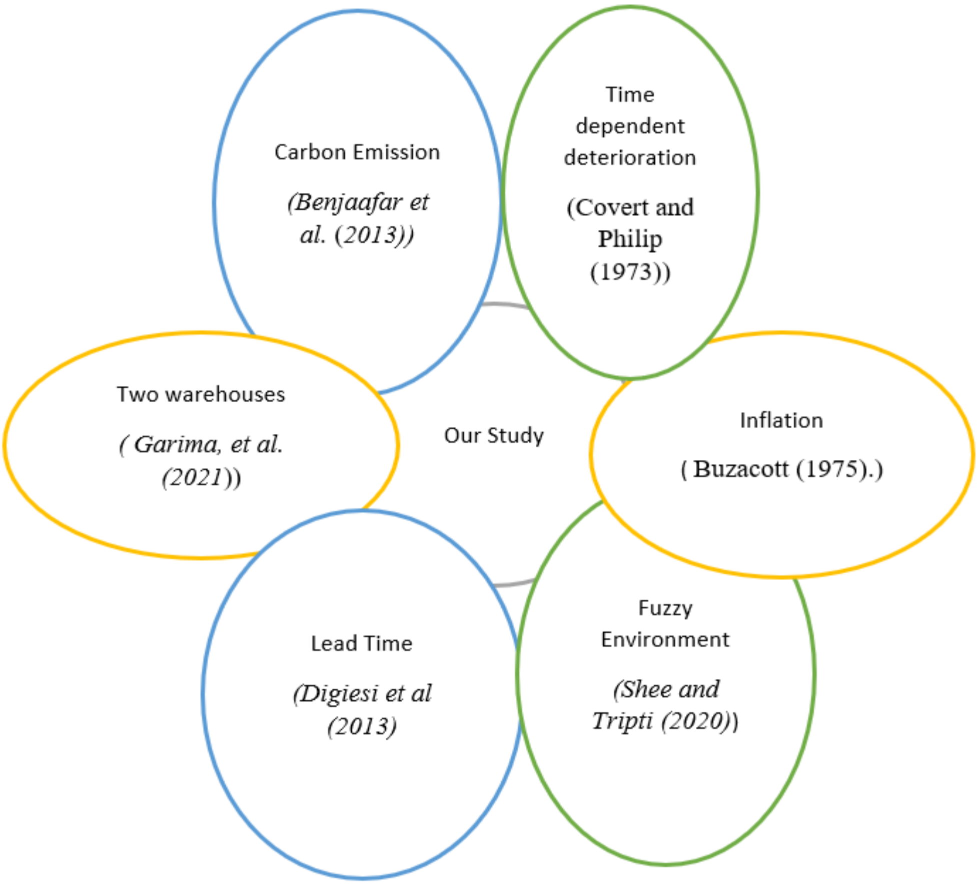

Figure 1. Pictorial Representation of Literature Review.

2. Assumptions and Notations

Assumptions made in this paper is as follows:

2.1 Assumptions

i. Lead time is a decision variable.
ii. Emission is considered in holding inventory, deteriorating items, ordering cost, and setup cost.
iii. The model is developed in a fuzzy environment.
iv. The retailer uses a two-warehouse facility.
v. Holding cost is time-varying for both the retailer and the supplier.
vi. Inflation is also allowed.
vii. Demand is exponentially increasing with time.
viii. Deterioration is time-dependent.
ix. Shortage is allowed for the retailer.

2.2 Notations

Notations	*Explanation*	*Notations*	*Explanations*
$D(t) = \alpha e^{\beta t}$	Time varying Demand rate which is increasing with time	$\theta_1 t$	Time dependent deterioration rate in rented warehouse.
Q	Order quantity	$\theta_2 t$	Time dependent deterioration rate in own warehouse.
W	Capacity of Own warehouse	$H_r(t)$	Retailer holding cost which increasing with time t, where $H_r(t) = C_0 t^n$ where C_0 and n are constant and C_0, n ≥2.
Q-W	Capacity of Rented warehouse	$\widetilde{H_r}$	Fuzzy holding cost per unit for the retailer
E_{hr}	Cost of Carbon emission due to holding items for retailer	$H_s(t)$	Supplier holding cost per unit where $H_s(t) = H_1 + H_2 t$ and $H_1, H_2 > 0$
$\widetilde{H_s}$(t)	Fuzzy Holding cost per unit for supplier	E_{hs}	Cost of Carbon emission due to holding items for suppliers
I_{Ro}	Retailer's inventory level in own warehouse at any time t	I_{Rr}	Retailer's inventory level in rented warehouse at any time t
I_s	Supplier's inventory level at any time t	C_1	Retailer's shortage cost per unit.
$\widetilde{C_1}$	Fuzzy shortage cost per unit for retailer.	C_2	Setup cost per each order for retailer
E_{sr}	Cost of carbon emission due to setup cost for retailer	$\widetilde{C_2}$	Fuzzy setup cost per each order for retailer
C_3	Setup cost per each order for supplier.	$\widetilde{C_3}$	Fuzzy setup cost per each order for supplier.
E_{ss}	Cost of Carbon emission due to setup cost for supplier.	C_4	Deterioration cost per unit for retailer
$\widetilde{C_4}$	Fuzzy deterioration cost per unit for retailer.	E_{dr}	Cost of carbon emission for deteriorating items for retailer
C_5	Deterioration cost per unit for supplier	$\widetilde{C_5}$	Fuzzy deterioration cost per unit for supplier
E_{ds}	Cost of carbon emission for deteriorating items for supplier	r	Rate of inflation
T	Order cycle time	L	Lead time of supplier
TC	Total cost of supply chain.	$\widetilde{TC}$	Fuzzy total cost of supply chain

3. Mathematical Modeling

In this supply chain, we have a single retailer and a single supplier. When the retailer's inventory becomes zero, he orders from the supplier immediately. The supplier always delivers the order after a lead time L. So, in that time, a shortage occurs for the retailer.

3.1 Mathematical Modeling for Retailer's

The retailer's initial inventory level is Q, in which W items are held in the own warehouse and Q-W items are held in the rented warehouse. At first, demand is fulfilled from the rented warehouse and after that from the own warehouse. Here, the inventory level depletes due to demand and deterioration. The solution of the governing differential equation describes the inventory level from 0 to t_1, t_1 to T-L and T-L to T with boundary conditions t =0, $I_{Ro}(0)$ = Q-W, $I_{Rr}(0)$ = W, t = t_1, $I_{Rr}(t_1)$ = 0, $I_{Ro}(t_1)$ =W-W_o, I_{Ro} (T-L)=0, I_{Ro}(T) =-B is given by

$$I_{Rr}(t) = a[(t_1 - t) + \frac{b(t_1^2 - t^2)}{2} + \frac{\theta_1(t_1^3 - t^3)}{6}]e^{\frac{-\theta_1 t^2}{2}} \qquad 0 \leq t \leq t_1 \qquad (1)$$

$$I_{Ro}(t) = We^{\frac{-\theta_2 t^2}{2}} \qquad 0 \leq t \leq t_1 \qquad (2)$$

$$I_{Ro}(t) = a\left[T - t + \frac{b((T-L)^2 - t^2)}{2} + \frac{\theta_2((T-L)^3 - t^3)}{6}\right]e^{\frac{-\theta_2 t^2}{2}} \qquad t_1 \leq t \leq T\text{-}L \qquad (3)$$

$$I_{Ro}(t) = a\left[T - t + \frac{b(T^2 - t^2)}{2}\right] - B \qquad T\text{-}L \leq t \leq T \qquad (4)$$

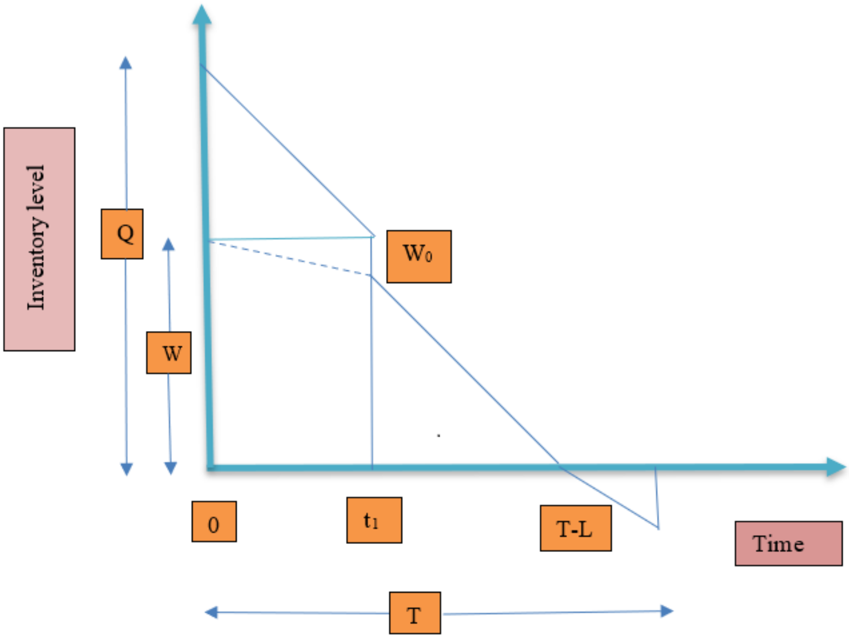

Figure 2. Describing the behavior of Retailer Inventory level.

Where B is stock out inventory and is given by

$$B = a[T - t + \frac{b(T^2 + L^2 - 2LT)}{2}] \qquad (5)$$

From using Continuity from equation (2) &(3) we obtain the value of W

$$W = a[((T - L) - t_1) + \frac{b}{2}[(T - L)^2 - t_1^2] + \frac{\theta_2}{2}[(T - L)^3 - t_1^3]] \qquad (6)$$

Inventory costs for retailer's are given as below

Cost of ordering = $C_2 + E_{sr}$ (7)

Holding Cost = $(C_0 + E_{hr})[\int_0^{t_1} t^n e^{-rt} I_{Rr}(t)dt + \int_0^{t_1} t^n e^{-rt} I_{Ro}(t)dt + \int_{t_1}^{T-L} t^n e^{-rt} I_{Ro}(t)dt]$

$$HC = (C_0 + E_{hr})[W\left[\frac{t_1^{n+1}}{(n+1)} - \frac{rt_1^{n+2}}{(n+2)} - \frac{\theta_2 t_1^{n+3}}{2(n+3)}\right] + a\left[\frac{t_1^{n+2}}{(n+1)(n+2)} + \frac{bt_1^{n+3}}{(n+1)(n+3)} + \frac{\theta_1 t_1^{n+4}}{2(n+1)(n+4)} - r\left[\frac{t_1^{n+3}}{(n+2)(n+3)} + \frac{bt_1^{n+4}}{(n+2)(n+4)} + \frac{\theta_1 t_1^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_1}{2}\left[\frac{t_1^{n+4}}{(n+3)(n+4)} + \frac{bt_1^{n+5}}{(n+3)(n+5)} + \frac{\theta_1 t_1^{n+6}}{2(n+3)(n+6)}\right]\right] + a\left[\frac{(T-L)^{n+2}}{(n+1)(n+2)} - \frac{(T-L)t_1^{n+1}}{(n+1)} + \frac{t_1^{n+2}}{(n+2)} + \frac{b(T-L)^{n+3}}{(n+1)(n+3)} + \frac{\theta_2(T-L)^{n+4}}{2(n+1)(n+4)} - r\left[\frac{(T-L)^{n+3}}{(n+2)(n+3)} + \frac{b(T-L)^{n+4}}{(n+2)(n+4)} + \frac{\theta_2(T-L)^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_2}{2}\left[\frac{(T-L)^{n+4}}{(n+3)(n+4)} + \frac{b(T-L)^{n+5}}{(n+3)(n+5)} + \frac{\theta_2(T-L)^{n+6}}{2(n+3)(n+6)}\right]\right] \qquad (8)$$

Shortage cost = $-C_1\int_{T-L}^{T} e^{-rt} I_{Ro}(t)dt$

$$= C_1[BT + ar[\frac{T^3}{6} + \frac{bT^4}{8} - \frac{T(T-L)^2}{2} + \frac{(T-L)^3}{3} - \frac{b}{2}\left(\frac{T^2(T-L)^2}{2} - \frac{(T-L)^4}{4}\right) - a\left[\frac{bT^3}{3} - \frac{T^2}{2} + LT + \frac{(T-L)^2}{2} - \frac{b}{2}\left(T^3 - T^2L - \frac{(T-L)^3}{3}\right)\right] - \frac{BrT^2}{2} - R(T-L) + \frac{Br(T-L)^2}{2}] \quad (9)$$

Deterioration Cost = $(C_4 + E_{dr})[a\theta_1\left[\frac{t_1^3}{6} + \frac{bt_1^4}{8} + \frac{\theta_1 t_1^5}{20} - r\left[\frac{t_1^4}{12} + \frac{bt_1^5}{15} + \frac{\theta_1 t_1^6}{36}\right] - \frac{\theta_1}{2}\left[\frac{t_1^5}{20} + \frac{bt_1^6}{24} + \frac{\theta_1 t_1^7}{56}\right]\right] + \theta_2 W\left[\frac{t_1^2}{2} - \frac{rt_1^3}{3} - \frac{\theta_2 t_1^4}{8}\right] + a\theta_2[\frac{(T-L)^3}{6} - \frac{(T-L)t_1^2}{2} + \frac{t_1^3}{3} + \frac{b(T-L)^4}{8} + \frac{\theta_2(T-L)^5}{20} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^2}{2} - \frac{t_1^4}{4}\right] - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{2} - \frac{t_1^5}{5}\right) - r[\frac{(T-L)^4}{12} - \frac{(T-L)t_1^3}{3} + \frac{t_1^4}{4} + \frac{b(T-L)^5}{15} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^3}{3} - \frac{t_1^5}{5}\right] + \frac{\theta_2(T-L)^6}{36} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{3} - \frac{t_1^6}{6}\right] - \theta_2\left[\frac{(T-L)^5}{40} - \frac{(T-L)t_1^4}{8} + \frac{t_1^5}{10} + \frac{b(T-L)^6}{48} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^4}{8} - \frac{t_1^6}{12}\right] + \frac{\theta_2(T-L)^7}{112} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^4}{8} - \frac{t_1^7}{14}\right]\right]]$ (10)

Average total Cost for retailer is given by

$TC_r(T, L) = \frac{1}{T}$ [Ordering cost +Holding Cost +Deterioration Cost + Shortage Cost]

$$TC_r(T,L) = \frac{1}{T}[(C_0 + E_{hr})[W\left[\frac{t_1^{n+1}}{(n+1)} - \frac{rt_1^{n+2}}{(n+2)} - \frac{\theta_2 t_1^{n+3}}{2(n+3)}\right] + a\left[\frac{t_1^{n+2}}{(n+1)(n+2)} + \frac{bt_1^{n+3}}{(n+1)(n+3)} + \frac{\theta_1 t_1^{n+4}}{2(n+1)(n+4)} - r\left[\frac{t_1^{n+3}}{(n+2)(n+3)} + \frac{bt_1^{n+4}}{(n+2)(n+4)} + \frac{\theta_1 t_1^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_1}{2}\left[\frac{t_1^{n+4}}{(n+3)(n+4)} + \frac{bt_1^{n+5}}{(n+3)(n+5)} + \frac{\theta_1 t_1^{n+6}}{2(n+3)(n+6)}\right]\right] + a\left[\frac{(T-L)^{n+2}}{(n+1)(n+2)} - \frac{(T-L)t_1^{n+1}}{(n+1)} + \frac{t_1^{n+2}}{(n+2)} + \frac{b(T-L)^{n+3}}{(n+1)(n+3)} + \frac{\theta_2(T-L)^{n+4}}{2(n+1)(n+4)} - r\left[\frac{(T-L)^{n+3}}{(n+2)(n+3)} + \frac{b(T-L)^{n+4}}{(n+2)(n+4)} + \frac{\theta_2(T-L)^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_2}{2}\left[\frac{(T-L)^{n+4}}{(n+3)(n+4)} + \frac{b(T-L)^{n+5}}{(n+3)(n+5)} + \frac{\theta_2(T-L)^{n+6}}{2(n+3)(n+6)}\right]\right] + C_1[BT + ar\left[\frac{T^3}{6} + \frac{bT^4}{8} - \frac{T(T-L)^2}{2} + \frac{(T-L)^3}{3} - \frac{b}{2}\left(\frac{T^2(T-L)^2}{2} - \frac{(T-L)^4}{4}\right) - a\left[\frac{bT^3}{3} - \frac{T^2}{2} + LT + \frac{(T-L)^2}{2} - \frac{b}{2}\left(T^3 - T^2L - \frac{(T-L)^3}{3}\right)\right] - \frac{BrT^2}{2} - R(T-L) + \frac{Br(T-L)^2}{2}\right] + (C_4 + E_{dr})[a\theta_1\left[\frac{t_1^3}{6} + \frac{bt_1^4}{8} + \frac{\theta_1 t_1^5}{20} - r\left[\frac{t_1^4}{12} + \frac{bt_1^5}{15} + \frac{\theta_1 t_1^6}{36}\right] - \frac{\theta_1}{2}\left[\frac{t_1^5}{20} + \frac{bt_1^6}{24} + \frac{\theta_1 t_1^7}{56}\right]\right] + \theta_2 W\left[\frac{t_1^2}{2} - \frac{rt_1^3}{3} - \frac{\theta_2 t_1^4}{8}\right] + a\theta_2[\frac{(T-L)^3}{6} - \frac{(T-L)t_1^2}{2} + \frac{t_1^3}{3} + \frac{b(T-L)^4}{8} + \frac{\theta_2(T-L)^5}{20} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^2}{2} - \frac{t_1^4}{4}\right] - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{2} - \frac{t_1^5}{5}\right) - r\left[\frac{(T-L)^4}{12} - \frac{(T-L)t_1^3}{3} + \frac{t_1^4}{4} + \frac{b(T-L)^5}{15} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^3}{3} - \frac{t_1^5}{5}\right] + \frac{\theta_2(T-L)^6}{36} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{3} - \frac{t_1^6}{6}\right] - \theta_2\left[\frac{(T-L)^5}{40} - \frac{(T-L)t_1^4}{8} + \frac{t_1^5}{10} + \frac{b(T-L)^6}{48} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^4}{8} - \frac{t_1^6}{12}\right] + \frac{\theta_2(T-L)^7}{112} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^4}{8} - \frac{t_1^7}{14}\right]\right]\right] + C2 + Esr] \quad (11)$$

3.2 Mathematical Modeling for supplier's

The supplier's inventory cycle starts at time t = 0, and the initial inventory level is Q. The inventory level decreases due to the joint effect of demand and deterioration. At time T-L, the supplier's inventory level becomes zero. The solution for the governing differential equation describes the inventory level with the boundary conditions: $t=0$, $I_s(0)=Q$, and at $t=T-L$, $I_s(T-L)=0$

$$I_s(t) = a[\left((T-L) - t\right) + \frac{b[(T-L)^2 - t^2]}{2} + \frac{\theta[(T-L)^3 - t^3]}{6}]e^{\frac{\theta t^2}{2}} \qquad 0 \le t \le T-L \qquad (12)$$

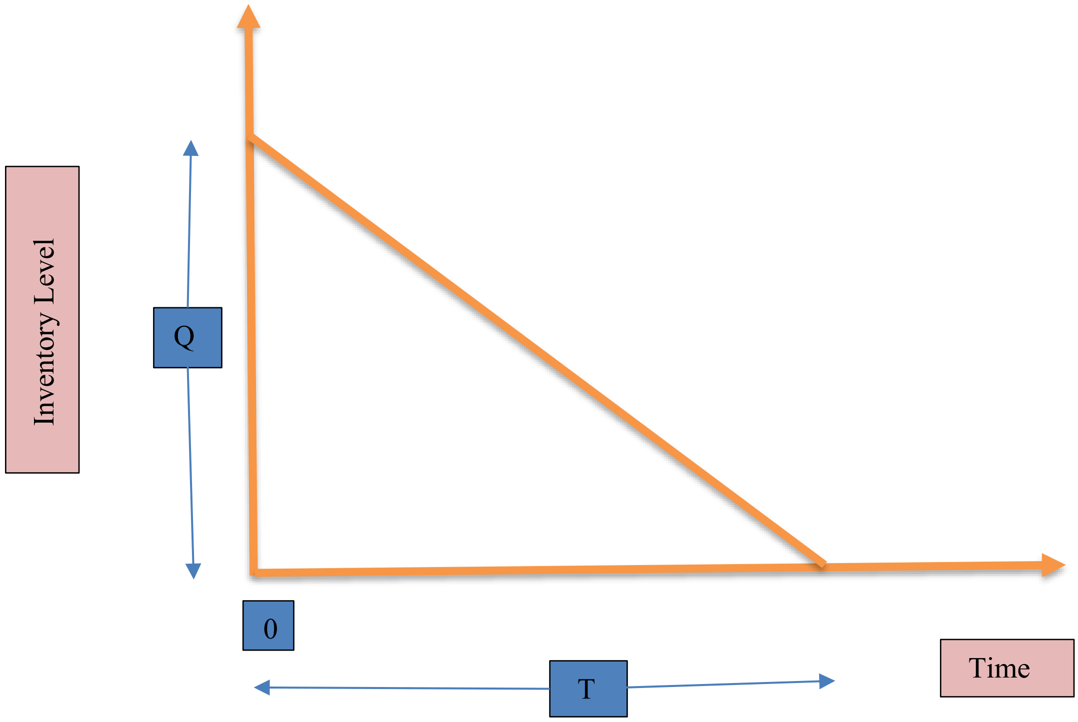

Figure 3. Describing the behavior of inventory level for supplier's.

And Order quantity is given by

$$Q = a[(T-L) + \frac{b(T-L)^2}{2} + \frac{\theta(T-L)^3}{6}] \qquad (13)$$

Inventory cost for supplier's is given by

Setup Cost = C_3+E_{ss} (14)

Holding Cost = $\int_0^{T-L}(h_1 + h_2 t + E_{hs})e^{-rt}I_s(t)$

$$=(h_1 + E_{hs})\left[\frac{(T-L)^2}{2} + \frac{b(T-L)^3}{3} + \frac{\theta(T-L)^4}{8} - r\left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20}\right] - \frac{\theta}{2}\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right]\right] + h_2 a[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20} - r\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right] - \frac{\theta}{2}\left[\frac{(T-L)^5}{20} + \frac{b(T-L)^6}{24} + \frac{\theta(T-L)^7}{56}\right]] \qquad (15)$$

Deterioration cost = $\int_0^{T-L}(C_5 + E_{ds})\, e^{-rt}I_s(t)dt$

$$= (C_5 + E_{ds})[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20} - r\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right] - \frac{\theta}{2}\left[\frac{(T-L)^5}{20} + \frac{b(T-L)^6}{24} + \frac{\theta(T-L)^7}{56}\right]] \qquad (16)$$

Therefore, the supplier's average total cost in cycle time T is

$TC_s(T, L) = \frac{1}{T}$ [Setup Cost + Holding Cost + Deterioration Cost]

$$TC_s(T, L) = \frac{1}{T}[(h_1 + E_{hs})\left[\frac{(T-L)^2}{2} + \frac{b(T-L)^3}{3} + \frac{\theta(T-L)^4}{8} - r\left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20}\right] - \frac{\theta}{2}\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right]\right] + h_2 a\left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20} - r\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right] - \right.$$

$\frac{\theta}{2}\left[\frac{(T-L)^5}{20}+\frac{b(T-L)^6}{24}+\frac{\theta(T-L)^7}{56}\right]\Bigg] + C_3 + E_{Ss} + (C_5+E_{ds})\Bigg[\frac{(T-L)^3}{6}+\frac{b(T-L)^4}{8}+\frac{\theta(T-L)^5}{20}-r\left[\frac{(T-L)^4}{12}+\frac{b(T-L)^5}{15}+\frac{\theta(T-L)^6}{36}\right]-\frac{\theta}{2}\left[\frac{(T-L)^5}{20}+\frac{b(T-L)^6}{24}+\frac{\theta(T-L)^7}{56}\right]\Bigg]\Bigg]$ (17)

Therefore, the total

TC(T, L , t1) =TCr (T, L, t1) + TCs(T, L)

$$\mathrm{TC}(T,L,t_1) = \frac{1}{T}\Bigg[(C_0+E_{hr})\Bigg[W\left[\frac{t_1^{n+1}}{(n+1)}-\frac{rt_1^{n+2}}{(n+2)}-\frac{\theta_2 t_1^{n+3}}{2(n+3)}\right]+a\Bigg[\frac{t_1^{n+2}}{(n+1)(n+2)}+\frac{bt_1^{n+3}}{(n+1)(n+3)}+\frac{\theta_1 t_1^{n+4}}{2(n+1)(n+4)}-r\left[\frac{t_1^{n+3}}{(n+2)(n+3)}+\frac{bt_1^{n+4}}{(n+2)(n+4)}+\frac{\theta_1 t_1^{n+5}}{2(n+2)(n+5)}\right]-\frac{\theta_1}{2}\left[\frac{t_1^{n+4}}{(n+3)(n+4)}+\frac{bt_1^{n+5}}{(n+3)(n+5)}+\frac{\theta_1 t_1^{n+6}}{2(n+3)(n+6)}\right]\Bigg]+a\Bigg[\frac{(T-L)^{n+2}}{(n+1)(n+2)}-\frac{(T-L)t_1^{n+1}}{(n+1)}+\frac{t_1^{n+2}}{(n+2)}+\frac{b(T-L)^{n+3}}{(n+1)(n+3)}+\frac{\theta_2(T-L)^{n+4}}{2(n+1)(n+4)}-r\left[\frac{(T-L)^{n+3}}{(n+2)(n+3)}+\frac{b(T-L)^{n+4}}{(n+2)(n+4)}+\frac{\theta_2(T-L)^{n+5}}{2(n+2)(n+5)}\right]-\frac{\theta_2}{2}\left[\frac{(T-L)^{n+4}}{(n+3)(n+4)}+\frac{b(T-L)^{n+5}}{(n+3)(n+5)}+\frac{\theta_2(T-L)^{n+6}}{2(n+3)(n+6)}\right]\Bigg] + C_1\Bigg[BT+ar\left[\frac{T^3}{6}+\frac{bT^4}{8}-\frac{T(T-L)^2}{2}+\frac{(T-L)^3}{3}-\frac{b}{2}\left(\frac{T^2(T-L)^2}{2}-\frac{(T-L)^4}{4}\right)-a\left[\frac{bT^3}{3}-\frac{T^2}{2}+LT+\frac{(T-L)^2}{2}-\frac{b}{2}\left(T^3-T^2L-\frac{(T-L)^3}{3}\right)\right]-\frac{BrT^2}{2}-R(T-L)+\frac{Br(T-L)^2}{2}\right]+(C_4+E_{dr})\Bigg[a\theta_1\left[\frac{t_1^3}{6}+\frac{bt_1^4}{8}+\frac{\theta_1 t_1^5}{20}-r\left[\frac{t_1^4}{12}+\frac{bt_1^5}{15}+\frac{\theta_1 t_1^6}{36}\right]-\frac{\theta_1}{2}\left[\frac{t_1^5}{20}+\frac{bt_1^6}{24}+\frac{\theta_1 t_1^7}{56}\right]\right]+\theta_2 W\left[\frac{t_1^2}{2}-\frac{rt_1^3}{3}-\frac{\theta_2 t_1^4}{8}\right]+a\theta_2\Bigg[\frac{(T-L)^3}{6}-\frac{(T-L)t_1^2}{2}+\frac{t_1^3}{3}+\frac{b(T-L)^4}{8}+\frac{\theta_2(T-L)^5}{20}-\frac{b}{2}\left[\frac{(T-L)^2t_1^2}{2}-\frac{t_1^4}{4}\right]-\frac{\theta_2}{6}\left[\frac{(T-L)^3t_1^3}{2}-\frac{t_1^5}{5}\right)-r\Bigg[\frac{(T-L)^4}{12}-\frac{(T-L)t_1^3}{3}+\frac{t_1^4}{4}+\frac{b(T-L)^5}{15}-\frac{b}{2}\left[\frac{(T-L)^2t_1^3}{3}-\frac{t_1^5}{5}\right]+\frac{\theta_2(T-L)^6}{36}-\frac{\theta_2}{6}\left[\frac{(T-L)^3t_1^3}{3}-\frac{t_1^6}{6}\right]-\theta_2\left[\frac{(T-L)^5}{40}-\frac{(T-L)t_1^4}{8}+\frac{t_1^5}{10}+\frac{b(T-L)^6}{48}-\frac{b}{2}\left[\frac{(T-L)^2t_1^4}{8}-\frac{t_1^6}{12}\right]+\frac{\theta_2(T-L)^7}{112}-\frac{\theta_2}{6}\left[\frac{(T-L)^3t_1^4}{8}-\frac{t_1^7}{14}\right]\right]\Bigg]\Bigg]\Bigg] + C2 + E_{Sr} + \left[(h_1+E_{hs})\left[\frac{(T-L)^2}{2}+\frac{b(T-L)^3}{3}+\frac{\theta(T-L)^4}{8}-r\left[\frac{(T-L)^3}{6}+\frac{b(T-L)^4}{8}+\frac{\theta(T-L)^5}{20}\right]-\frac{\theta}{2}\left[\frac{(T-L)^4}{12}+\frac{b(T-L)^5}{15}+\frac{\theta(T-L)^6}{36}\right]\right]+h_2 a\left[\frac{(T-L)^3}{6}+\frac{b(T-L)^4}{8}+\frac{\theta(T-L)^5}{20}-r\left[\frac{(T-L)^4}{12}+\frac{b(T-L)^5}{15}+\frac{\theta(T-L)^6}{36}\right]-\frac{\theta}{2}\left[\frac{(T-L)^5}{20}+\frac{b(T-L)^6}{24}+\frac{\theta(T-L)^7}{56}\right]\right]\right] + C_3 + E_{Ss} + (C_5+E_{ds})\left[\frac{(T-L)^3}{6}+\frac{b(T-L)^4}{8}+\frac{\theta(T-L)^5}{20}-r\left[\frac{(T-L)^4}{12}+\frac{b(T-L)^5}{15}+\frac{\theta(T-L)^6}{36}\right]-\frac{\theta}{2}\left[\frac{(T-L)^5}{20}+\frac{b(T-L)^6}{24}+\frac{\theta(T-L)^7}{56}\right]\right]\Bigg] \quad (18)$$

Fuzzy Model for Supply chain

In this paper we use triangular fuzzy number to fuzzified the parameters C_0, C_1, C_2, C_3, C_4, C_5, h_1, h_2, E_{hs}, E_{Sr}, E_{ds}, E_{Ss}, E_{hr}, E_{dr},

Let $\widetilde{C_0}=(x_0,y_0,z_0)$, $\widetilde{C_1}=(x_1,y_1,z_1)$, $\widetilde{C_2}=(x_2,y_2,z_2)$, $\widetilde{C_3}=(x_3,y_3,z_3)$, $\widetilde{C_4}=(x_4,y_4,z_4)$, $\widetilde{C_5}=(x_5,y_5,z_5)$, $\widetilde{h_1}=(\alpha_1,\alpha_2,\alpha_3)$, $\widetilde{h_2}=(\beta_1,\beta_2,\beta_3)$, $\widetilde{E_{hs}}=(\gamma_1,\gamma_2,\gamma_3)$, $\widetilde{E_{hr}}=(a_1,b_1,c_1)$, $\widetilde{E_{sr}}=(a_2,b_2,c_2)$, $\widetilde{E_{dr}}=(a_3,b_3,c_3)$, $\widetilde{E_{Ss}}=(a_4,b_4,c_4)$, $\widetilde{E_{ds}}=(a_5,b_5,c_5)$

Then the total average cost for fuzzy supply chain model

$$\widetilde{TC} = \frac{1}{T}\left[(\widetilde{C_0} + \widetilde{E_{hr}})R + \widetilde{C_1}S + (\widetilde{C_2} + \widetilde{E_{sr}}) + (\widetilde{C_4} + \widetilde{E_{dr}})U + (\widetilde{h_1} + \widetilde{E_{hs}})X + \widetilde{h_2}Y + (\widetilde{C_3} + \widetilde{E_{Ss}}) + (\widetilde{C_5} + \widetilde{E_{ds}})Z\right]$$

Where:

$$R = [W\left[\frac{t_1^{n+1}}{(n+1)} - \frac{rt_1^{n+2}}{(n+2)} - \frac{\theta_2 t_1^{n+3}}{2(n+3)}\right] + a\left[\frac{t_1^{n+2}}{(n+1)(n+2)} + \frac{bt_1^{n+3}}{(n+1)(n+3)} + \frac{\theta_1 t_1^{n+4}}{2(n+1)(n+4)} - r\left[\frac{t_1^{n+3}}{(n+2)(n+3)} + \frac{bt_1^{n+4}}{(n+2)(n+4)} + \right.\right.$$

$$\left.\left.\frac{\theta_1 t_1^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_1}{2}\left[\frac{t_1^{n+4}}{(n+3)(n+4)} + \frac{bt_1^{n+5}}{(n+3)(n+5)} + \frac{\theta_1 t_1^{n+6}}{2(n+3)(n+6)}\right]\right] + a\left[\frac{(T-L)^{n+2}}{(n+1)(n+2)} - \frac{(T-L)t_1^{n+1}}{(n+1)} + \frac{t_1^{n+2}}{(n+2)} + \frac{b(T-L)^{n+3}}{(n+1)(n+3)} + \right.$$

$$\left.\frac{\theta_2 (T-L)^{n+4}}{2(n+1)(n+4)} - r\left[\frac{(T-L)^{n+3}}{(n+2)(n+3)} + \frac{b(T-L)^{n+4}}{(n+2)(n+4)} + \frac{\theta_2 (T-L)^{n+5}}{2(n+2)(n+5)}\right] - \frac{\theta_2}{2}\left[\frac{(T-L)^{n+4}}{(n+3)(n+4)} + \frac{b(T-L)^{n+5}}{(n+3)(n+5)} + \frac{\theta_2 (T-L)^{n+6}}{2(n+3)(n+6)}\right]\right]$$

$$S = [BT + ar[\frac{T^3}{6} + \frac{bT^4}{8} - \frac{T(T-L)^2}{2} + \frac{(T-L)^3}{3} - \frac{b}{2}\left(\frac{T^2(T-L)^2}{2} - \frac{(T-L)^4}{4}\right) - a\left[\frac{bT^3}{3} - \frac{T^2}{2} + LT + \frac{(T-L)^2}{2} - \right.$$

$$\left.\frac{b}{2}\left(T^3 - T^2L - \frac{(T-L)^3}{3}\right)\right] - \frac{BrT^2}{2} - R(T-L) + \frac{Br(T-L)^2}{2}]$$

$$U = [a\theta_1\left[\frac{t_1^3}{6} + \frac{bt_1^4}{8} + \frac{\theta_1 t_1^5}{20} - r\left[\frac{t_1^4}{12} + \frac{bt_1^5}{15} + \frac{\theta_1 t_1^6}{36}\right] - \frac{\theta_1}{2}\left[\frac{t_1^5}{20} + \frac{bt_1^6}{24} + \frac{\theta_1 t_1^7}{56}\right]\right] + \theta_2 W\left[\frac{t_1^2}{2} - \frac{rt_1^3}{3} - \frac{\theta_2 t_1^4}{8}\right] +$$

$$a\theta_2[\frac{(T-L)^3}{6} - \frac{(T-L)t_1^2}{2} + \frac{t_1^3}{3} + \frac{b(T-L)^4}{8} + \frac{\theta_2(T-L)^5}{20} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^2}{2} - \frac{t_1^4}{4}\right] - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{2} - \frac{t_1^5}{5}\right) - r[\frac{(T-L)^4}{12} - \frac{(T-L)t_1^3}{3} +$$

$$\frac{t_1^4}{4} + \frac{b(T-L)^5}{15} - \frac{b}{2}\left[\frac{(T-L)^2 t_1^3}{3} - \frac{t_1^5}{5}\right] + \frac{\theta_2(T-L)^6}{36} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^3}{3} - \frac{t_1^6}{6}\right] - \theta_2\left[\frac{(T-L)^5}{40} - \frac{(T-L)t_1^4}{8} + \frac{t_1^5}{10} + \frac{b(T-L)^6}{48} - \right.$$

$$\left.\frac{b}{2}\left[\frac{(T-L)^2 t_1^4}{8} - \frac{t_1^6}{12}\right] + \frac{\theta_2(T-L)^7}{112} - \frac{\theta_2}{6}\left[\frac{(T-L)^3 t_1^4}{8} - \frac{t_1^7}{14}\right]\right]]$$

$$X = \left[\frac{(T-L)^2}{2} + \frac{b(T-L)^3}{3} + \frac{\theta(T-L)^4}{8} - r\left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20}\right] - \frac{\theta}{2}\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right]\right]$$

$$Y = \left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20} - r\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right] - \frac{\theta}{2}\left[\frac{(T-L)^5}{20} + \frac{b(T-L)^6}{24} + \frac{\theta(T-L)^7}{56}\right]\right]$$

$$Z = \left[\frac{(T-L)^3}{6} + \frac{b(T-L)^4}{8} + \frac{\theta(T-L)^5}{20} - r\left[\frac{(T-L)^4}{12} + \frac{b(T-L)^5}{15} + \frac{\theta(T-L)^6}{36}\right] - \frac{\theta}{2}\left[\frac{(T-L)^5}{20} + \frac{b(T-L)^6}{24} + \frac{\theta(T-L)^7}{56}\right]\right]$$

The fuzzy total cost is given by

$$\widetilde{TC} = (TC_1, TC_2, TC_3)$$

$$TC_1 = \frac{1}{T}[(x_0 + a_1)R + x_1 S + (x_4 + a_4)U + (x_2 + a_2) + (\alpha_1 + \gamma_1)X + \beta_1 Y + (x_3 + a_3) + (x_5 + a_5)Z]$$

$$TC_2 = \frac{1}{T}[(y_0 + b_1)R + y_1 S + (y_4 + b_4)U + (y_2 + b_2) + (\alpha_2 + \gamma_2)X + \beta_2 Y + (y_3 + b_3) + (y_5 + b_5)Z]$$

$$TC_3 = \frac{1}{T}[(z_0 + c_1)R + z_1 S + (z_4 + c_4)U + (z_2 + c_2) + (\alpha_3 + \gamma_3)X + \beta_3 Y + (z_3 + c_3) + (z_5 + c_5)Z]$$

We find the total Average fuzzy cost of supply chain by two Method

i. By Signed Distance Method

$$TC_s = \frac{1}{4}[TC_1 + 2TC_2 + TC_3]$$

$$TC_s = \frac{1}{4T}[[(x_0 + a_1) + 2(y_0 + b_1) + (z_0 + c_1)R + (x_1 + 2y_1 + z_1)S + [(x_4 + a_4) + 2(y_4 + b_4) + (z_4 + c_4)]U + [(x_2 + a_2) + 2(y_2 + b_2) + (z_2 + c_2)] + [(\alpha_1 + \gamma_1) + 2(\alpha_2 + \gamma_2) + (\alpha_3 + \gamma_3)]X + [\beta_1 + 2\beta_2 + \beta_3]Y + [(x_3 + a_3) + 2(y_3 + b_3) + (z_3 + c_3)] + [(x_5 + a_5) + 2(y_5 + b_5) + (z_5 + c_5)]Z]$$

ii. By Graded Mean Integration Method

$$TC_s = \frac{1}{6}[TC_1 + 4TC_2 + TC_3]$$

$$TC_s = \frac{1}{4T}[[(x_0 + a_1) + 2(y_0 + b_1) + (z_0 + c_1)R + (x_1 + 2y_1 + z_1)S + [(x_4 + a_4) + 2(y_4 + b_4) + (z_4 + c_4)]U + [(x_2 + a_2) + 2(y_2 + b_2) + (z_2 + c_2)] + [(\alpha_1 + \gamma_1) + 2(\alpha_2 + \gamma_2) + (\alpha_3 + \gamma_3)]X + [\beta_1 + 2\beta_2 + \beta_3]Y + [(x_3 + a_3) + 2(y_3 + b_3) + (z_3 + c_3)] + [(x_5 + a_5) + 2(y_5 + b_5) + (z_5 + c_5)]Z]$$

4. Algorithm to solve the Mathematical Model

The total cost equation has three independent variables, L, t_1, and T, according to the suggested model. The following processes are used to optimize the total cost equation and determine the values of all the independent parameters.

Step. 1 Determine the first-order partial derivative for each independent variable. $\frac{\partial TC}{\partial L}, \frac{\partial TC}{\partial t_1}$ *and* $\frac{\partial TC}{\partial T}$

Step.2 Equates the first order partial derivatives to zero and solve the value of L, t_1, and T

Step.3 Now calculate the second-order partial derivatives w.r.t. all the independent variable like $\frac{\partial^2 TC}{\partial L^2}, \frac{\partial^2 TC}{\partial T^2}$ *and* $\frac{\partial^2 TC}{\partial t_1^2}$

Step.4

Now, form a Hessian matrix as follows

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial L^2} & \frac{\partial^2 TC}{\partial L \partial t_1} & \frac{\partial^2 TC}{\partial L \partial T} \\ \frac{\partial^2 TC}{\partial t_1 \partial L} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial t \partial L} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{bmatrix}$$

Step.5

Find H_1, H_2 and H_3, where H_1, H_2, and H_3 denote the first principal minor, second principal minor and third principal minor respectively. If det (H_1)>, 0, det $(H_2)>0$ and det $(H_3)>0$, then the matrix is positive definite matrix, and TC is called convex function.

5. Numerical Illustration

We carried out the Numerical illustration by using the software MATHEMATICA 12.0. Based on the previous research these parameters are taken in appropriate unit are as follows

For crisp Model

θ_1=0.02, θ_2 = 0.01, r =0,1, a =40, b=0.5, n=2, C_0=2, C_1=8, C_2=100, C_4=5, C_5=7, C_3=200, h_1=2, h_2=0.2, E_{sr} = 0.01, E_{hr} =0.10, E_{dr} =1, E_{ss}= 1.101, E_{hs} =1.010, E_{ds} = 1.01

We get TC = 275.909, T =1.88082, L= 0.591262, t_1 = 0.017014

For Fuzzy Model

By signed distance Method

$\widetilde{C_0} = (1, 2, 3)$, $\widetilde{C_1} = (7,8,9)$, $\widetilde{C_2} = (95, 100,105)$, $\widetilde{C_3} = (195, 200,205)$, $\widetilde{C_4} = (4, 5, 6)$, $\widetilde{C_5} = (6, 7, 8)$, $\widetilde{h_1} = (0.1, 0.2, 0.3)$, $\widetilde{h_2} = (0.05, 0.1,0.15)$, θ_1=0.02, θ_2 = 0.01, r =0,1, a =40, b=0.5, n=2, $\widetilde{E_{hs}} = (1,1.010,1.020)$, $\widetilde{E_{hr}} = (0.009,0.01,0.11)$, $\widetilde{E_{sr}} = (0.9,1,1.1)$, $\widetilde{E_{dr}} = (0.009,0.01,0.011)$, $\widetilde{E_{Ss}} = (1,1.101,1.102)$, $\widetilde{E_{ds}} = (1,1.010,1.02)$

We get TC =234.328, L= 0.521217, T=1.99532, t_1= 0.620019

By Graded Mean Integration Method

$\widetilde{C_0} = (1, 2, 3)$, $\widetilde{C_1} = (7,8,9)$, $\widetilde{C_2} = (95, 100,105)$, $\widetilde{C_3} = (195, 200,205)$, $\widetilde{C_4} = (4, 5, 6)$, $\widetilde{C_5} = (6, 7, 8)$, $\widetilde{h_1} = (0.1, 0.2, 0.3)$, $\widetilde{h_2} = (0.05, 0.1,0.15)$, θ_1=0.02, θ_2 = 0.01, r =0,1, a =40, b=0.5, n=2, $\widetilde{E_{hs}} = (1,1.010,1.020)$, $\widetilde{E_{hr}} = (0.009,0.01,0.11)$, $\widetilde{E_{sr}} = (0.9,1,1.1)$, $\widetilde{E_{dr}} = (0.009,0.01,0.011)$, $\widetilde{E_{Ss}} = (1,1.101,1.102)$, $\widetilde{E_{ds}} = (1,1.010,1.02)$

We get TC =221.282, L= 0.501281, T=2.0442, t_1= 0.6494
From this we find the optimal solution for fuzzy model by graded Mean integration Method.

6. Convexity for Crisp Model/ Fuzzy Model

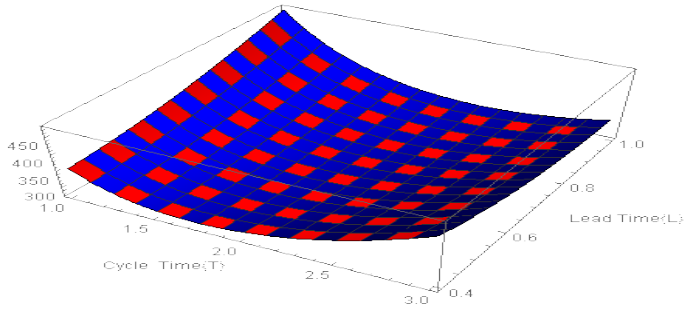

Figure 4. Convexity of Crisp Model.

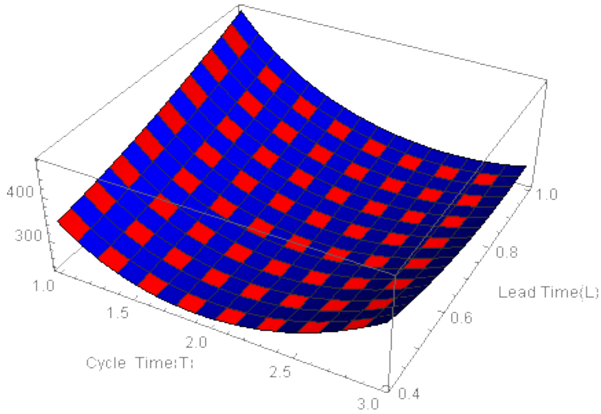

Figure 5. Convexity for Fuzzy Model (By Signed Distance Method).

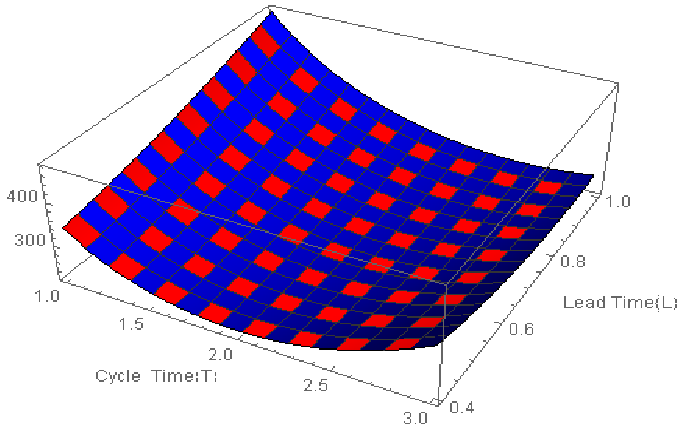

Figure 6. Convexity for Fuzzy Model by Graded Mean Distance Method.

7. Sensitivity Analysis

The sensitivity is carried out by the variation of different parameters r, b, a, n, θ, taking variation in on parameter and keeping all other parameter fixed. Now we find the sensitivity of the optimal total cost to changes in the values of different parameters of model.

Figure 7. Impact of a on TC for Crisp Model.

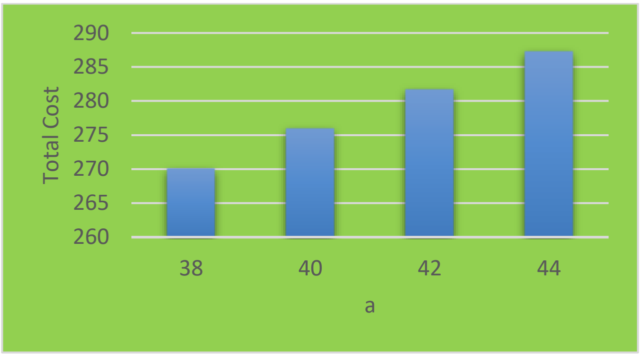

Figure 8. Impact of a on TC_s (Fuzzy Model).

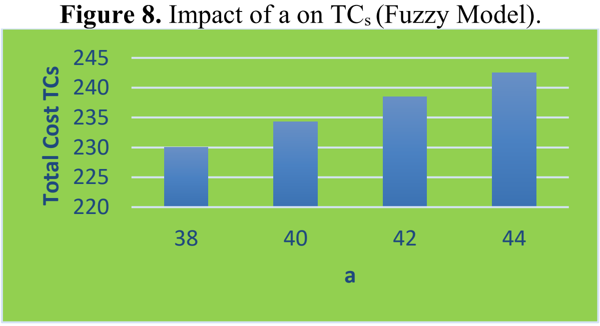

Figure 9. Impact of a on TC_G (Fuzzy Model).

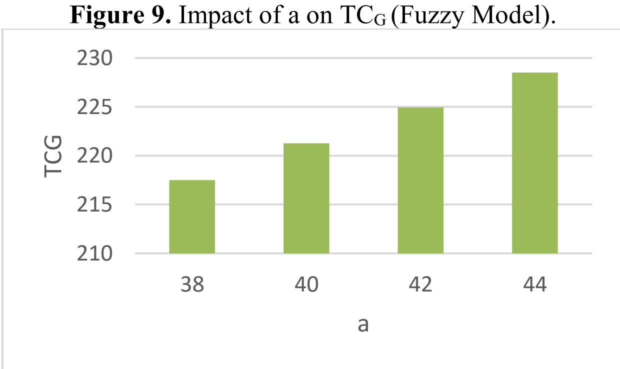

Figure 10. Impact of r on TC.

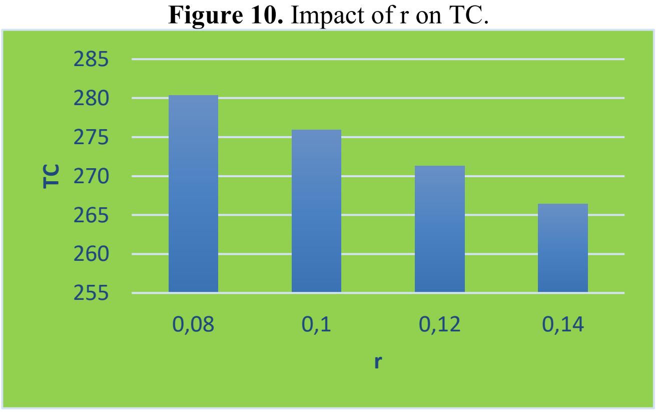

Figure 11. Impact of r on TC_S.

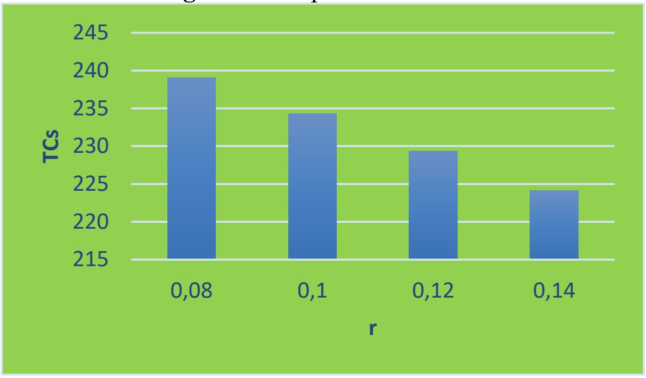

Figure 12. Impact of r on TC_G.

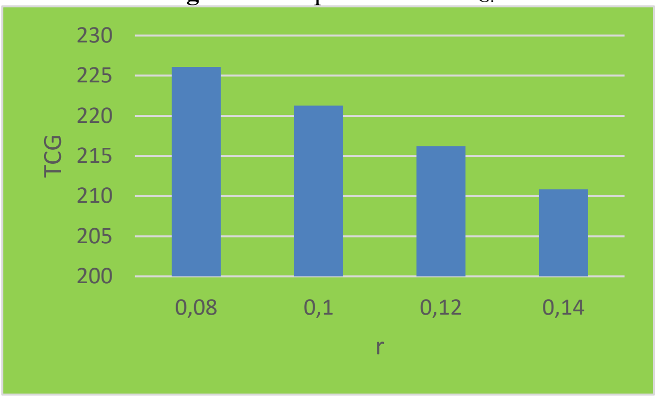

Figure 13. Impact of b on TC.

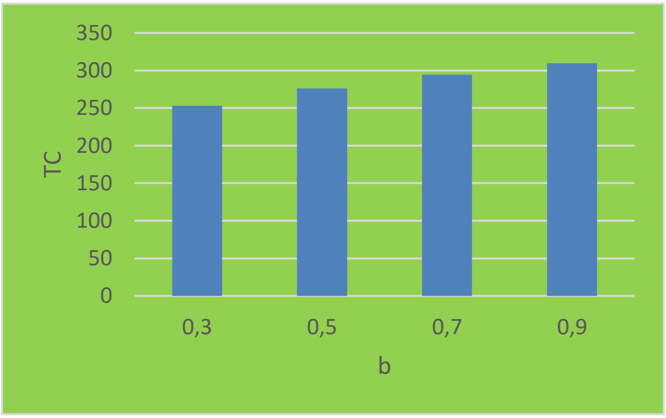

Figure 14. Impact of b on TC_s.

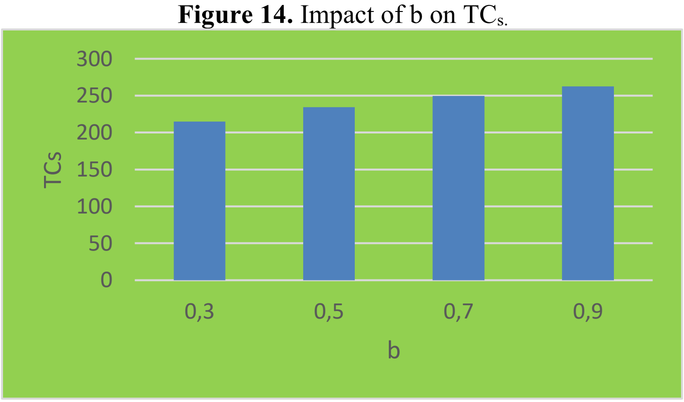

Figure 15. Impact of b on TC_G.

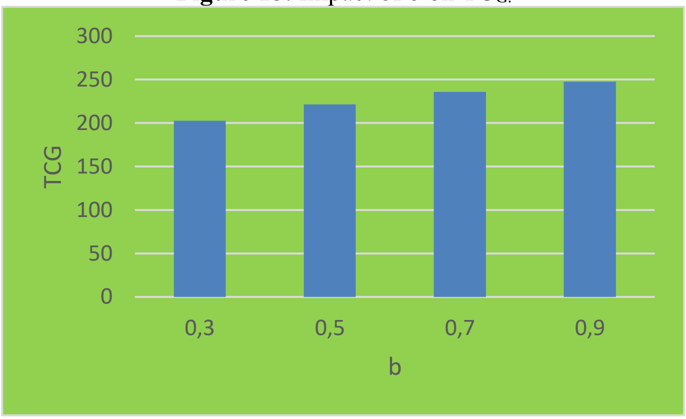

Figure 16. Impact of n on TC.

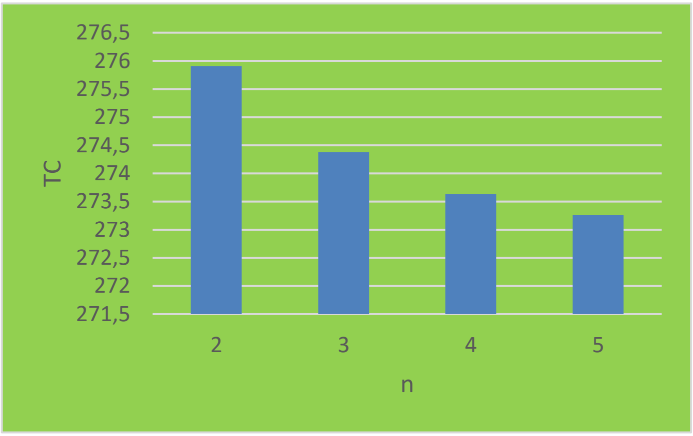

Figure 17. Impact of n on TC_S.

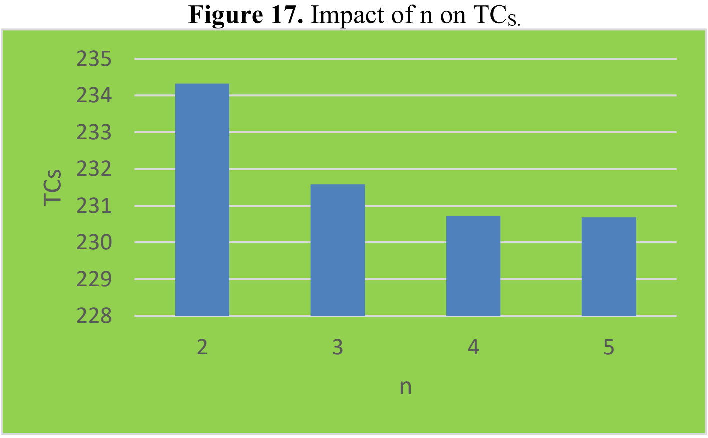

Figure 18. Impact of n on TC_G.

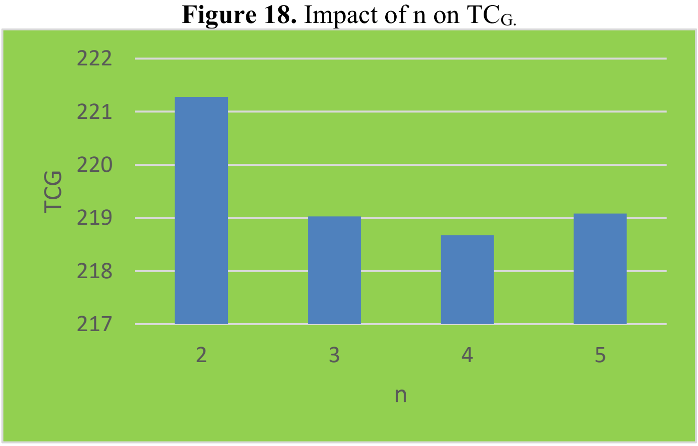

Table 2.1 Sensitivity Analysis

Parameters	Change Value	Crisp Model			Fuzzy Model by Signed Distance Method			Fuzzy Model by Graded Mean Integration method		
		T	L	TC	T	L	TC_s	T	L	TC_G
a	38	1.91722	0.6042	270.015	2.0280	0.534	230.066	2.0764	0.514	217.495
	40	1.88082	0.5912	275.909	1.9953	0.521	234.328	2.0442	0.501	221.282
	42	1.84671	0.5792	281.652	1.9646	0.509	238.468	2.014	0.490	224.955
	44	1.81466	0.5679	287.254	1.9358	0.498	242.497	1.9856	0.479	228.523
b	0.3	2.13432	0.6988	252.931	2.2324	0.632	214.804	2.2834	0.614	202.736
	0.5	1.88082	0.5913	275.909	1.9953	0.521	234.328	2.0442	0.501	221.282
	0.7	1.7255	0.5286	294.176	1.8480	0.456	249.638	1.8964	0.435	235.669
	0.9	1.61529	0.4890	309.684	1.7426	0.412	262.518	1.7908	0.391	247.688
r	0.08	1.84341	0.5793	280.36	1.9536	0.505	239.033	1.99973	0.4831	226.08
	0.1	1.88082	0.5913	275.909	1.9953	0.521	234.328	2.0442	0.50120	221.282
	0.12	1.92146	0.604	271.271	2.0408	0.538	229.383	2.09276	0.5203	216.218
	0.14	1.96587	0.617	266.427	2.0907	0.556	224.167	2.14622	0.5404	210.852
n	2	1.88082	0.5913	275.909	1.9953	0.521	234.328	2.0442	0.5013	221.282
	3	1.87892	0.5887	274.381	1.9604	0.516	231.577	1.99336	0.4971	219.025
	4	1.8735	0.5876	273.638	1.9262	0.515	230.725	1.94809	0.4962	218.674
	5	1.86616	0.5872	273.263	1.8955	0.515	230.679	1.90979	0.4965	219.079
θ	0.001	1.88168	0.5912	275.814	1.9963	0.5210	234.199	2.04521	0.5010	221.137
	0.002	1.88082	0.5913	275.909	1.9953	0.5212	234.328	2.0442	0.5012	221.282
	0.003	1.87996	0.5914	276.003	1.9944	0.5214	234.457	2.04318	0.5014	221.426
	0.004	1.87911	0.5916	276.097	1.9935	0.5216	234.585	Negative	0.967	Negative

8. Observations

The following observations are based on sensitivity analysis:

1. It is observed that on increasing the parameters a and b, the total cost increases for both crisp and fuzzy models. Whereas, on increasing these two parameters, the cycle length and lead time decrease. Also, we observed that the total cost for both models is moderately sensitive to variations in "a". On the other hand, the total cost is highly sensitive to variations in the value of "b".

2. As the deterioration parameter increases, the total cost and lead time increase for both models, but cycle length decreases.

3. As the parameter n increases, the total cost, lead time, and cycle length decrease.

4. As the parameter r increases, the total cost decreases, but cycle length and lead time increase.

9. Conclusion

In this study, we developed an optimal approach for a two-echelon supply chain model, focusing on the retailer's suppliers. The retailer's merchandise is stored in two warehouses. Our objective is to determine the optimal total cost, cycle duration, and supplier lead time. We calculate the total cost using three methods: first, for the crisp model; second, for the fuzzy model using the signed distance method; and third, using the Graded Mean Integration method. The numerical illustration shows that the total cost for the fuzzy model, determined by the Graded Mean Integration method, is the lowest. This study can be further expanded to include a stochastic model.

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Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: S. SINGH, D. SINGH, **Data curation: Formal analysis:** A. GAUR, **Investigation:** S. SINGH, D. SINGH **Methodology:DS Software: DS, Supervision:** S. SINGH, **Validation: Visualization: Writing - original draft:** D. SINGH, A. GAUR, **Writing - review and editing:** D. SINGH DS, A. GAUR.

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