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ARTICLE

Residual analysis for Minimum Guarantee transformation regression model with application

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Abstract
This paper proposes a new regression model in which the response variable follows the Minimum Guarantee Akash distribution. The model parameters are estimated using the maximum likelihood estimation method, and a Monte Carlo simulation study is performed to investigate the finite-sample behavior of the estimators in terms of bias, mean square error, and consistency. Model adequacy is evaluated through residual diagnostics, while influential observations are detected using Cook's distance. The proposed Minimum Guarantee Akash regression model is further compared with Weibull and Gamma regression models using common goodness-of-fit measures such as AIC, BIC, and HQIC. The findings demonstrate that the Minimum Guarantee Akash regression model provides a better fit and greater flexibility for modeling lifetime and reliability data. Owing to its adaptability, the model can be effectively applied in economics, biological sciences, survival analysis, public health, engineering reliability, and risk assessment.
Keywords: Definition of Minimum Guarantee Akash distribution, Minimum Guarantee Akash Regression Model, Maximum Likelihood Estimation, Residual Analysis, Deviance Residuals, Martingale Residuals.

1. Introduction

A variety of probability distributions have been widely used to model data arising in many applied fields, including economics, biological sciences, mortality and recovery studies, health research, reliability analysis, metrology, medicine, engineering, insurance, and finance. In recent years, increasing attention has been given to the development of flexible regression models based on specialized probability distributions. This interest is mainly driven by the need to better represent skewness, kurtosis, and other complex characteristics commonly observed in real datasets.

Several regression models based on extended and reparameterized distributions have been introduced in the statistical literature. For example, Altun *et al.* (2020) proposed the unit-improved second-degree Lindley distribution for both inferential and regression purposes. Cordeiro *et al.* (2015) developed the log-generalized modified Weibull regression model, while Silva *et al.* (2008) introduced the Log-Burr XII regression model for lifetime data analysis. Chanasriphum *et al.* (2019) presented the log-beta generalized Weibull regression model, which offers greater flexibility in survival studies. More recently, Mazucheli *et al.* (2021) proposed a quantile regression model for bounded data using the Birnbaum–Saunders distribution.

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Additional studies have further expanded this area of research. Korkmaz *et al.* (2021) introduced the unit Burr-XII quantile regression model, while Khan (2021) proposed the exponentiated Weibull regression model. Júnior *et al.* (2021) developed the log-generalized inverse Weibull regression model, and Granzotto *et al.* (2018) presented the transmuted Weibull regression model. Ocloo *et al.* (2023) extended the Burr XII distribution with important applications in regression analysis. Other notable contributions include the Gumbel–Burr XII regression model by Al-Aqtash *et al.* (2021), the Zografos–Balakrishnan Burr XII regression model by Altun *et al.* (2023), the Unit-Chen quantile regression model by Korkmaz *et al.* (2023), and the unit generalized half-normal quantile regression model by Mazucheli *et al.* (2023). In addition, Benkhelifa (2022) introduced the log-beta power Muth regression model, which further enriched the family of flexible lifetime regression models.

In this context, the Minimum Guarantee Akash distribution appears as an important and flexible model for analyzing non-negative data. This distribution is derived by combining the Akash distribution with the minimum guarantee concept, which enhances its ability to model different shapes of hazard functions as well as varying levels of skewness and kurtosis. Due to its flexibility, the Minimum Guarantee Akash distribution has attracted attention in reliability engineering, survival analysis, and risk assessment studies. Compared with traditional lifetime distributions, it can effectively model both light-tailed and heavy-tailed data, making it suitable for datasets with heterogeneous variation. Parameter estimation is commonly carried out using the maximum likelihood method, while model performance is often evaluated by comparing it with well-known competing models such as the Weibull and gamma distributions.

Although many flexible regression models based on lifetime distributions have been proposed, most of the existing studies mainly focus on Weibull- and Burr-type distributions, with limited attention given to regression models based on the Akash distribution. Furthermore, comparative studies involving the Minimum Guarantee Akash model and classical regression models remain relatively scarce. To address this gap, the present study proposes a Minimum Guarantee Akash regression model and provides a comprehensive comparison with classical gamma and Weibull regression models. The aim is to demonstrate the superiority of the proposed model in terms of goodness of fit, flexibility, and robustness when modeling lifetime and reliability data.

The remainder of this paper is organized as follows. Section (2) introduces the Minimum Guarantee Akash distribution. Section (3) discusses its main mathematical properties. Section (4) presents the proposed regression model based on the Minimum Guarantee Akash distribution. Section (5) focuses on residual analysis techniques. Section (6) describes the simulation study, while Section (7) provides an application using real data. Finally, Section (8) presents the main conclusions of the study.

2. Materials and Methods

2.1 Introduction

Numerous approaches have been introduced in the statistical literature to construct new probability distributions, typically by adopting well-known distributions as baseline models. For example, Gupta *et al.* (1998) proposed the exponentiated exponential distribution by applying a transformation to the cumulative distribution function of a baseline distribution. Rahman *et al.* (2018) developed the cubic transmuted family as a generalization of the quadratic transmuted distribution. The DUS transformation was introduced by Kumar *et al.* (2016), whereas the sine-based SS transformation was proposed by Kumar *et al.* (2015). Furthermore, Kumar *et al.* (2017) utilized the MG transmutation map to generalize the exponential distribution. In the present work, the MG transmutation map introduced by Kumar *et al.* (2017) is adopted to construct a new distribution that extends the exponentiated gamma Lindley model.

2.1.1 The Minimum Guarantee transformtion

The Minimum Guarantee transmutation, introduced by Kumar *et al.* (2017), constitutes a flexible mechanism for extending classical probability distributions and constructing more adaptable models. By incorporating additional shape parameters, this transformation enables the generalization of well-

known distributions, such as the exponential and gamma, thereby improving their ability to capture complex data patterns. Earlier contributions include the exponentiated exponential distribution proposed by Gupta *et al.* (1998) through transformations of baseline cumulative distribution functions. In a related vein, Rahman *et al.* (2018) developed the cubic transmuted family as a generalization of the quadratic transmuted distribution. Other notable transformations include the sine-based SS transformation suggested by Kumar *et al.* (2015) and the DUS transformation introduced by Kumar *et al.* (2016).

In the present study, the Minimum Guarantee transmutation plays a central role and is employed to construct an extended version of the exponentiated gamma Lindley distribution, thereby increasing its flexibility and broadening its applicability to a wide range of datasets. The Minimum Guarantee transmutation is formally defined as follows:

$$G(x) = \exp(1 - \frac{1}{F(x)}) \qquad x > 0 \qquad (1)$$

and probability density function (pdf) is defined by

$$g(x) = f(x).\frac{\exp(1-\frac{1}{F(x)})}{[F(x)]^2} \qquad x > 0 \qquad (2)$$

where F(x) and f(x) are, respectively, the base distribution's cdf and pdf.

2.2 Minimum Guarantee Akash (MGA) Distribution

This section presents the newly proposed MG Gamma Lindley (MGGL) distribution. The corresponding cumulative distribution function (CDF) and probability density function (PDF) are derived and discussed. Let X denote a continuous random variable following the MG akash distribution; the CDF and PDF characterize its probabilistic behavior.

$$G(x) = \exp(1 - \frac{\exp(\beta x)}{\exp(\beta x) \quad -\left[1 + \frac{\beta x(\beta x + 2)}{\beta^2 + 2}\right]}) \qquad (3)$$

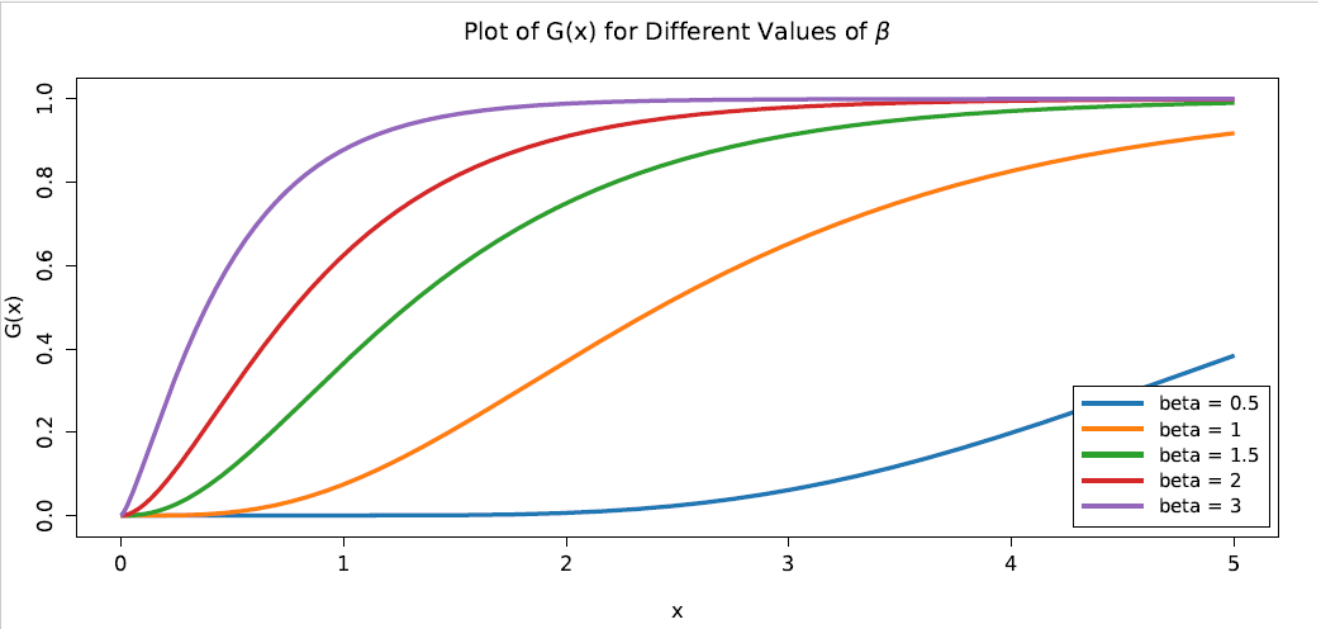


Figure 1. CDF of Minimum Guarantee Akash) Distribution at different value.

The associated probability density function takes the following form

$$pdf\ \ g(x) = K_1.\frac{K_2}{K_3} \qquad ,\beta > 0 \qquad x > 0 \tag{4}$$

where

$$K_1 = \frac{\beta^3}{\beta^2 + 2}\,[x^2 + 1].\exp(-\beta x) \tag{5}$$

$$,K_2 = exp(1 - \frac{exp(\beta x)}{exp(\beta x) \ -\left[1+\frac{\beta x(\beta x+2)}{\beta^2+2}\right]}$$

$$K_3 = \left[1 - \left[1 + \frac{\beta x(\beta x + 2)}{\beta^2 + 2}\right]\ \exp(-\beta x)\ \right]^2$$

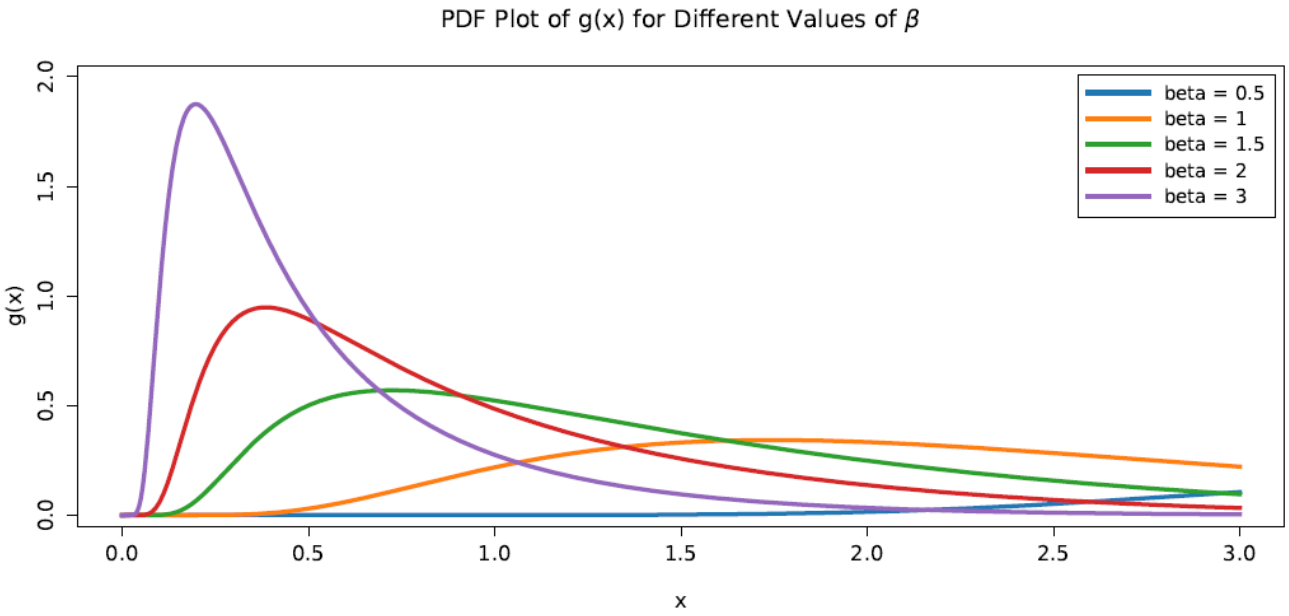


Figure 2. pdf of Minimum Guarantee Akash Distribution at different value.

2.3 Some Mathematical Properties for Minimum Guarantee akash

In this part, intereutecd in finding the expressions for the survival function, hazard rate function, and reverse-hazard rate function of the Minimum Guarantee gamma Lindley-distribution.

2.3.1 Survival Function

Let X be a continuous random variable with a cumulative distribution function on the interval [0,∞] called F(x). The following is its survival or reliability function

$$S(x,\alpha,\beta) = 1 - \exp(1 - \frac{\exp(\beta x)}{\exp(\beta x) \ -\left[1+\frac{\beta x(\beta x+2)}{\beta^2+2}\right]}) \tag{6}$$

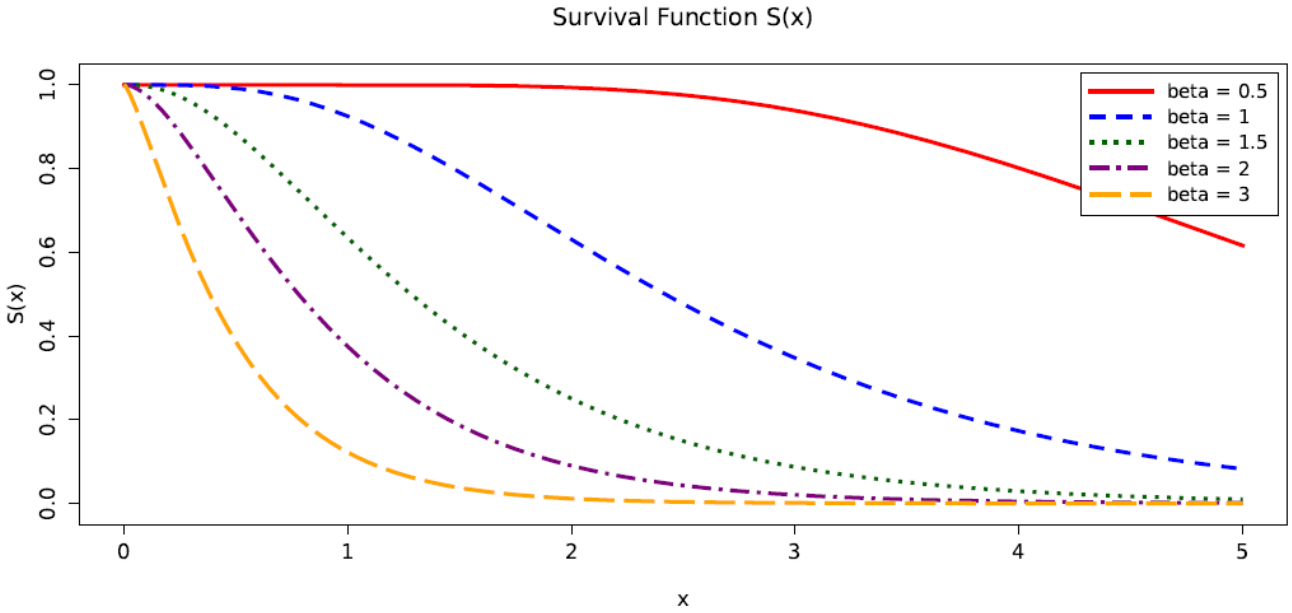


Figure 3. Survival Function of Minimum Guarantee Akash (MGA) Distribution at different value.

The figure displays the survival function (S(x)) of the Minimum Guarantee gamma–Lindley distribution for different combinations of (alpha) and (beta). In all panels, the survival curves decrease monotonically from near one to zero, confirming the validity of the model for lifetime data. Larger values of (\alpha) lead to a faster initial decline, indicating a higher early-failure rate, while increasing (beta) results in heavier tails and slower long-term decay. These plots demonstrate the flexibility of the model in representing diverse survival patterns.

2.3.2 Hazard rate function

A continuous random variable X with pdf f (x) and cdf F (x) is defined as having the hazard rate function (also known as the failure rate function) h(x) .

$$h(x,\alpha,\beta) = \frac{K_1 . \frac{K_2}{K_3}}{1 - \exp(1 - \frac{\exp(\beta x)}{\exp(\beta x) \quad - \left[1 + \frac{\beta x(\beta x + 2)}{\beta^2 + 2}\right]})} \tag{7}$$

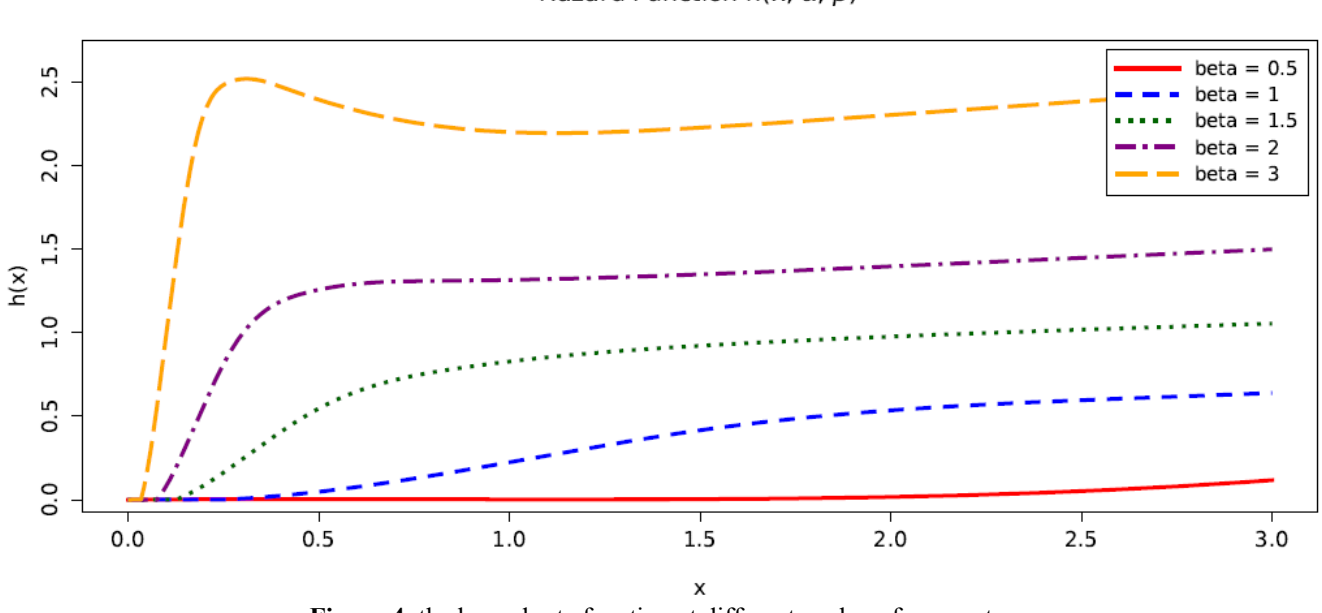


Figure 4. the hazard rate function at differente value of parameter.

2.4 Regression Model for Minimum Guarantee Akash

Regression modeling was widely used in fields such as economics, engineering, and the natural sciences to describe relationships among variables. Although the classical normal regression model was commonly applied, its assumptions were often restrictive and frequently violated in practice. To address this issue, Nelder and Wedderburn (1972) introduced the Generalized Linear Model (GLM), which allowed response variables to follow non-normal distributions. Later, McCullagh and Nelder (1989) extended the GLM framework to cover a wide range of distributions, including gamma, beta, Bernoulli, and Poisson..

Following this line of research, the regression framework was extended to the MGakash distribution. When the location and scale parameters were adopted to characterize the random variable X, the corresponding regression model was obtained as follow Let X be a random variable having the akash density function and let random variable $y = \sigma log x$, $\beta = exp(-\frac{\mu}{\sigma})$, for $y \in R$ Differentiating the hypothesis we get the following $dy = \frac{\sigma dx}{x}$ implies to $\frac{dy}{dx} = \frac{\sigma}{x}$ but the Jacob equal $\frac{dx}{dy} = \frac{x}{\sigma}$., the density function of Y can be written as

$$g(y, \mu, \sigma) = g^{-1}(x).|J|\,. \tag{8}$$

$$pdf\ \ g(x) = K_1.\frac{K_2}{K_3} \qquad ,\beta > 0 \qquad x > 0 \tag{9}$$

where

$$K_1 = \frac{e^{\left(\frac{y-\mu}{\sigma}\right)}}{\left[2 + e^{-\left(\frac{2\mu}{\sigma}\right)}\right]\sigma}.\left[e^{-\frac{2\mu}{\sigma}} + e^{\left(\frac{2(y-\mu)}{\sigma}\right)}\right].exp\left(-e^{\frac{y-\mu}{\sigma}}\right) \tag{10}$$

$$,K_2 = exp(1 - \frac{exp\left(e^{\frac{y-\mu}{\sigma}}\right)}{exp\left(e^{\frac{y-\mu}{\sigma}}\right) - \left[1 + \frac{e^{\frac{y-\mu}{\sigma}}(e^{\frac{y-\mu}{\sigma}} + 2)}{exp(-\frac{2\mu}{\sigma}) + 2}\right]}$$

$$K_3 = \left[1 - \left[1 + \frac{e^{\frac{y-\mu}{\sigma}}(e^{\frac{y-\mu}{\sigma}} + 2)}{exp(-\frac{2\mu}{\sigma}) + 2}\right] \exp\left(-e^{\frac{y-\mu}{\sigma}}\right)\right]^2$$

$$S(x, \alpha, \beta) = 1 - \exp\left(1 - \frac{\exp\left(e^{\frac{y-\mu}{\sigma}}\right)}{\exp\left(e^{\frac{y-\mu}{\sigma}}\right) - \left[1 + \frac{e^{\frac{y-\mu}{\sigma}}\left(e^{\frac{y-\mu}{\sigma}} + 2\right)}{\exp\left(-\frac{2\mu}{\sigma}\right) + 2}\right]}\right) = 1 - \exp(K_2) \tag{11}$$

We suggest a new log-location-scale regression model based on the MGAkash density function. Let Y_i be the response variable following the MGAkash distribution, and $x = (x_1, x_2, \ldots . x_n)$ be the independent variable. The regression model is defined as

$$y_i = x^{T\beta} + \sigma z_i \tag{12}$$

where σ is the random error with PDF in equation (10) $\beta = (\beta_0, \beta_1, \ldots\ldots\ldots\ldots \beta_n)$, are unknown parameters . y_i is the location of $\mu = x^T\beta$ and the location vector $\mu = (\mu_1, \ldots\ldots\ldots\ldots \mu_n)$

2.5 Estimation of the Model Parameters

Maximum Likelihood Estimation (MLE) is a well-established technique for estimating the parameters of a statistical model by maximizing the likelihood function, which quantifies the agreement between the model and the observed data. For a given sample, MLE determines the parameter values that render the observed outcomes most likely. Owing to its favorable theoretical properties—such as consistency, efficiency, and invariance—MLE is extensively applied in areas including economics, biological sciences, and machine learning..

For the right-censored lifetime data, we have $t_i = \min(f_i, c_i)$, where f_i is the lifetime and C_i *i*s the censoring time, then, we have $y_i = \log(t_i)$ for the ith individual $i = 1, \ldots, n$. If we have random sample with n observations $(y_1, , x_1)$,..., $((y_n, , x_n)$,

$$\delta_i = \begin{cases} 1 & for y_i = \log(t_i) \\ 0 & for y_i = log(c_i) \end{cases}$$
$$l = \prod_{i=0}^{n} [f(y)]^{\delta_i} . [s(y)]^{(1-\delta_i)} \tag{13}$$

. The function of log-likelihood is given

$$\mathbf{L}(\theta) = \sum_{i \in F}^{n} \delta_i \, . logf(y_i) + \sum_{i \in s}^{n} (1 - \delta_i) \log(s(y_i)) \tag{14}$$

Where $f(y_i)$ is the density function of Akash distribution and $s(y_i)$ is survival function

$$\mathbf{L}(\boldsymbol{\theta}) = K_4 + K_6.$$

$$K_4 = \sum_{i \in F}^{n} \delta_i \, . logg(y_i) \qquad K_6 = \sum_{i \in s}^{n} (1 - \delta_i) \log(s(y_i)) \tag{15}$$

$$K_4 = \sum_{i \in F}^{n} \delta_i \, . logK_1 . \frac{K_2}{K_3} = \sum_{i=1}^{n} \delta_i logK_1 + \sum_{i=1}^{n} \delta_i \log K_2 - \sum_{i=1}^{n} \delta_i \log k_3 \tag{16}$$

$$K_6 = \textstyle\sum_{i \in s}^{n} (1 - \delta_i) \log(1 - exp(K_2))$$

regression by minimizing the log-likelihood function

$$\frac{\partial l(\theta)}{\partial \beta_0} = \frac{\partial K_4}{\partial \beta_0} + \frac{\partial k_5}{\partial \beta_0} \qquad \frac{\partial l(\theta)}{\partial \sigma} = \frac{\partial K_4}{\partial \sigma} + \frac{\partial k_5}{\partial \sigma} .., \frac{\partial l(\theta)}{\partial \beta_1} = \frac{\partial K_4}{\partial \beta_1} + \frac{\partial k_5}{\partial \beta_1} , \tag{17}$$

By solving the previous equations, I use the R software to obtain the parameter values.

2.6 Residual analysis

We require a method to verify assumptions after constructing the model. Several types of residuals can be used for this purpose, including the martingale residual, deviance component residual, Cox residual, Pearson residual, and Cox-Snell residual. Below, we explain each of these concepts in detail.

2.6.1 The Martingale Residual

The martingale residual is commonly used in the context of counting processes. These residuals are asymmetric, with values ranging from a minimum of negative infinity (-∞) to a maximum of +1. The martingale residual is defined as: [continue with the definition].

$$\delta_i = r_{Mi} = \begin{cases} 1 - log[1 - \exp(K_2)] & \delta_i = 1 \\ log\,[1 - exp(K_2)] & \delta_i = 0 \end{cases} \tag{18}$$

2.6.2 Deviance component Residual

This residue was suggested to make the martingale residual more symmetric around zero. The deviance component for the parametric regression model is given

$$\hat{r}_{Di} = sign\,(\hat{r}_{Mi})\,\{-2\,[\hat{r}_{Mi} + log\,(1-\hat{r}_{Mi})]\}^{1/2} \quad (19)$$

Where $\boldsymbol{r_{Mi}}$ is the martingale residual. * sign () function is a function that drives the (+***1***) values if the argument is positive and (-***1***) is negative (Yashpal Singh Raghavet al .2025)

2.6.3 Pearson Residuals

Are used to detect outliers It depends on the idea of subtracting the mean and dividing by the standard deviation. The Pearson Residual knows the bounded regression of the inverse of the exponential distribution as follows

$$r_i = \frac{y_{i-}\mu_i}{\sqrt{var(y)}} \quad (20)$$

2.6.4 Cox-Snell Residual

Is a diagnostic tool used in survival analysis to assess the goodness-of-fit of a model, particularly in the Cox proportional hazards model. It is calculated using the estimated cumulative baseline hazard function and the covariates of an individual. If the model fits well, the Cox-Snell residuals should follow a standard exponential distribution with a mean of 1. They are commonly used in model diagnostics by plotting them against the cumulative hazard function, where a straight line indicates a good model fit. This residual helps evaluate how well the model describes the observed data, making it a fundamental tool in academic survival analysis. Defined as follows.

$$e_i = -ln[1 - F(y_i, \beta)] \quad (21)$$

3. Sensitivity Analysis

Refers to the degree to which the outputs or predictions of a statistical or mathematical model respond to changes in its inputs, parameters, or assumptions. Assessing sensitivity is crucial for understanding the robustness and reliability of a model, as it helps identify which parameters have the most influence on the results. This analysis is widely applied in regression, survival, and simulation studies to guide model improvement and decision-making. There are generally three main types of sensitivity in statistical and mThis measures the proportion of the total output variance attributed to β_i

3.1 Generalized Cook's Distance (GCD)

The generalized Cook's distance assesses the influence of a single observation on estimates of parameters by calculating changes in these parameter estimates whenever a particular observation is removed..

$$GCD = \left(\hat{\theta} - \hat{\theta}_{\iota}\right)^{T}\left[Var\left(\hat{\theta}\right)\right]^{-1}\left(\hat{\theta} - \hat{\theta}_{\iota}\right) \quad (22)$$

3.2 Likelihood Displacement (LD)

Likelihood Displacement assesses the influence of an observation through the change in the log-likelihood function when that observation is removed.

$$LD_i = 2\left[L\left(\hat{\theta}\right) - L\left(\hat{\theta}_{\iota}\right)\right] \quad (23)$$

4.Results and Discussion

4.1 Simulation study

Simulation study is given to evaluate the performance of coefficients for the proposed regression

Model, According to and following steps using Monte Carlo Simulation

4.2 Model Specification

$\boldsymbol{Let}\ \mathbf{Y_I} = \boldsymbol{\sigma Log X_I}$ where $\boldsymbol{X_I}$ follows the MG Akash distribution with parameter:

$$\boldsymbol{\mu = \beta_0 + \beta_1 . x_1} \qquad \boldsymbol{\beta = exp(-\frac{\beta_0 + \beta_1 . x_1}{\sigma})},$$

4.3 Simulation Design

4.3.1 Parameter Scenarios

- Scenario 1: β0=2, β1=0.5, σ=0.5
- Scenario 2: β0=3.9, β1=0.7, σ=0.3
- Scenario 3: β0=4.0, β1=0.7, σ=0.2
- Sample Sizes n=20.100,200,300
- Censoring Levels Low censoring ≈ 20% Moderate censoring ≈ 50% High censoring ≈ 80%

4.3.2 Simulation Algorithm

For each scenario and each combination of n and censoring level:

- Generate covariates: xi ~N(0,1)
- Compute linear predictor: $\boldsymbol{\mu = \beta_0 + \beta_1 . x_1}$
- Compute parameter: $\boldsymbol{\beta_i = exp(-\frac{\beta_0 + \beta_1 . x_1}{\sigma})}$,
- Generate uniform random variables: Ui~ Uniform(0,1)
- Solve for $\boldsymbol{X_I}$ using inverse CDF $\boldsymbol{U_i = F_X(x_i, \beta_i)}$
- Transform data: $\mathbf{Y_I} = \boldsymbol{\sigma Log X_I}$
- Generate censoring variable: $\boldsymbol{C_i} \sim \mathbf{Uniform(0, \tau)}$
- Estimate parameters using MLE: 4.3.5-Repeat for R=1000 replications.

- computed Abs bias ,MSE and RMSE where $\boldsymbol{bais} = \frac{1}{N}\sum_{i=1}^{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta})\boldsymbol{MSE} = \frac{1}{N}\sum_{i=1}^{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta})^2$ $\boldsymbol{RMSE} = \sqrt{\boldsymbol{MSE}}$

4.4 Result Summlation for simple MG Akash Regression Model Case 1

4.4.1 Low Censoring Level

Table 1. Bias , MSE and RMSE for Parameter under Censoring Level (δ = 0.20)

Parameter	20	100	200
b0_Bias	0.038345	0.017841	0.017257
b0_AbsBias	0.038345	0.017841	0.017257
b0_MSE	0.012523	0.002191	0.001237
b0_RMSE	0.111906	0.046811	0.035175
b1_Bias	0.003824	0.002645	0.002638
b1_AbsBias	0.003824	0.002645	0.002638
b1_MSE	0.006344	0.001068	0.000495
b1_RMSE	0.079651	0.032676	0.022254
sigma_Bias	-0.045575	-0.022387	-0.020152
sigma_AbsBias	0.045575	0.022387	0.020152
sigma_MSE	0.009996	0.001785	0.001064
sigma_RMSE	0.099982	0.042250	0.032615
Mean_AIC	11.9116	102.4590	200.9817
Mean_BIC	14.8987	110.2745	210.8766
Mean_HQIC	12.4947	105.6220	204.9860

The results in Table 1 show that the **MG Akash Regression Model** performs very well under low censoring (δ = 0.20). As the sample size increases, Bias, MSE, and RMSE decrease for all

parameters, indicating accurate, stable, and efficient estimation. Even at small samples, errors remain relatively low, confirming good finite-sample performance. Overall, the model is highly reliable and provides precise estimates when censoring is

4.4.2 Moderate Censoring Level

Table 2. Bias ,MSE and RMSE OF parameter Model under Moderate Censoring Level (δ = 0.50)

Parameter	20	100	200
b0_Bias	0.045899	0.018649	0.016129
b0_AbsBias	0.045899	0.018649	0.016129
b0_MSE	0.014836	0.002345	0.001359
b0_RMSE	0.121802	0.048426	0.036869
b1_Bias	0.005060	0.002778	0.002713
b1_AbsBias	0.005060	0.002778	0.002713
b1_MSE	0.011448	0.001532	0.000833
b1_RMSE	0.106997	0.039138	0.028865
sigma_Bias	-0.051913	-0.023870	-0.019749
sigma_AbsBias	0.051913	0.023870	0.019749
sigma_MSE	0.020923	0.002878	0.001748
sigma_RMSE	0.144649	0.053646	0.041813
Mean_AIC	1.1266	116.6205	230.9464
Mean_BIC	4.1138	124.4360	240.8413
Mean_HQIC	1.7097	119.7836	234.9507

Table 2 show under moderate censoring (δ = 0.50), the **MG Akash regression model** shows slightly higher Bias and MSE, especially for small samples, due to information loss. However, as the sample size increases, RMSE and MSE decrease, confirming improved accuracy and consistency. The estimates of β_1 remain relatively stable, and model selection criteria indicate a reasonable fit, demonstrating that the model remains robust and reliable under moderate censoring.

4.4.3.4 High Censoring

Table 3. Bias ,MSE and RMSE for the parameter of Akash Regression Model under High Censoring

Parameter	20	100	200
b0_Bias	0.127820	0.027223	0.023061
b0_AbsBias	0.127820	0.027223	0.023061
b0_MSE	0.299274	0.005491	0.002524
b0_RMSE	0.547059	0.074098	0.050238
b1_Bias	0.064039	0.006875	0.006002
b1_AbsBias	0.064039	0.006875	0.006002
b1_MSE	0.214896	0.005723	0.002393
b1_RMSE	0.463569	0.075652	0.048915
sigma_Bias	-0.052117	-0.029987	-0.028488
sigma_AbsBias	0.052117	0.029987	0.028488
sigma_MSE	0.142768	0.017090	0.006801
sigma_RMSE	0.377847	0.130730	0.082469
Mean_AIC	-1.6921	82.2203	168.1084
Mean_BIC	1.2951	90.0358	178.0034
Mean_HQIC	-1.1090	85.3833	172.1127

Table 3 shows that under high censoring (δ = 0.80), the **MG Akash Regression Model** experiences larger estimation errors at small sample sizes (n = 20), especially for σ, due to significant information loss. However, as the sample size increases to 100 and 200, Bias and MSE decrease notably, indicating improved accuracy and stability. Overall, despite the challenges of heavy censoring, the mG Akash model maintains its consistency and performs effectively when larger samples are available.

4.5 Result Summlation for simple MG Akash Regression Model Case 2

4.5.1 Low Censoring (20%)

Table 4. Bias and MSE of the MG Akash Regression Model under Low Censoring

Parameter	n = 20	n = 100	n = 200
b0_Bias	0.02820	0.01061	0.01032
b0_AbsBias	0.02820	0.01061	0.01032
b0_MSE	0.01112	0.00154	0.00078
b0_RMSE	0.10546	0.03923	0.02787
b1_Bias	0.00604	0.00335	0.00262
b1_AbsBias	0.00604	0.00335	0.00262
b1_MSE	0.00377	0.00058	0.00027
b1_RMSE	0.06140	0.02403	0.01633
sigma_Bias	-0.02991	-0.01127	-0.01122
sigma_AbsBias	0.02991	0.01127	0.01122
sigma_MSE	0.00735	0.00103	0.00055
sigma_RMSE	0.08571	0.03213	0.02335
Mean_AIC	-72.73	-15.68	-37.63
Mean_BIC	-69.74	-7.86	-27.73
Mean_HQIC	-72.15	-12.51	-33.62

The table 4 shows that for the **MG Akash regression model** under moderate censoring (δ = 0.20), estimation accuracy improves as the sample size increases. Bias, MSE, and RMSE for all parameters (b_0, b_1, and σ) decrease steadily, indicating more precise and stable estimates, while the slight negative bias of σ diminishes with larger samples. Overall, the results confirm that the model performs efficiently and achieves better fit and reliability as the sample size grows.

4.5.2 Moderate Censoring (50%)

Table 5. Bias MSE of parameter Model under Moderate Censoring Level (δ = 0.50)

Parameter	n = 20	n = 100	n = 200
b0_Bias	0.05004	0.01413	0.01171
b0_AbsBias	0.05004	0.01413	0.01171
b0_MSE	0.01605	0.00193	0.00109
b0_RMSE	0.12667	0.04395	0.03298
b1_Bias	0.01991	0.01003	0.00860
b1_AbsBias	0.01991	0.01003	0.00860
b1_MSE	0.01357	0.00141	0.00078
b1_RMSE	0.11649	0.03752	0.02795
sigma_Bias	-0.03498	-0.00922	-0.00594
sigma_AbsBias	0.03498	0.00922	0.00594
sigma_MSE	0.01606	0.00170	0.00094
sigma_RMSE	0.12674	0.04128	0.03066
Mean_AIC	-75.47	13.90	26.21

Parameter	n = 20	n = 100	n = 200
Mean_BIC	-72.48	21.72	36.10
Mean_HQIC	-74.89	17.07	30.21

Table 5 summarizes the performance of the **MG Akash Regression Model** under moderate censoring (δ = 0.50). As the sample size increases from 20 to 200, the bias, absolute bias, MSE, and RMSE for all parameters (b0, b1, and σ) decrease steadily, indicating improved estimation accuracy and stability. The negative bias of σ becomes smaller, showing reduced underestimation with larger samples. Overall, the results confirm that the mG Akash model becomes more efficient with larger sample sizes despite the presence of censoring, while AIC, BIC, and HQIC reflect changes in model fit across different sample sizes.

4.5.3 High Censoring (80%)

Table 6. Bias MSE and RMSE of parameter Model under High Censoring Level

Parameter	n = 20	n = 100	n = 200
b0_Bias	0.29549	0.05736	0.03934
b0_AbsBias	0.29549	0.05736	0.03934
b0_MSE	0.84686	0.02152	0.00483
b0_RMSE	0.92025	0.14668	0.06952
b1_Bias	0.18986	0.04123	0.03374
b1_AbsBias	0.18986	0.04123	0.03374
b1_MSE	0.55820	0.01316	0.00534
b1_RMSE	0.74713	0.11472	0.07310
sigma_Bias	-0.02122	0.00689	0.01235
sigma_AbsBias	0.02122	0.00689	0.01235
sigma_MSE	0.09313	0.00954	0.00457
sigma_RMSE	0.30517	0.09765	0.06760
Mean_AIC	-45.02	23.70	51.87
Mean_BIC	-42.03	31.51	61.77
Mean_HQIC	-44.44	26.86	55.88

The table 6 shows that the **MG Akash Regression Model** improves significantly as the sample size increases. Bias, MSE, and RMSE for all parameters (b_0, b_1, and σ) decrease notably, indicating higher accuracy, consistency, and stability. While AIC, BIC, and HQIC increase with sample size due to likelihood effects, the overall results confirm that the model becomes more reliable and efficient with larger samples.

4.6 Result Summlation for simple MG Akash Regression Model Case 3

4.6.1 Low Censoring (δ = 0.20)

Table 7. Bias MSE of parameter Model under Low Censoring

Measure	20	100	200
b0_Bias	0.0321	0.00437	0.00131
b0_AbsBias	0.0321	0.00437	0.00202
b0_MSE	0.02667	0.00104	0.00044
b0_RMSE	0.16331	0.03231	0.02106
b1_Bias	0.00829	0.00185	0.00135
b1_AbsBias	0.00829	0.00185	0.00195
b1_MSE	0.00915	0.00029	0.00013
b1_RMSE	0.09564	0.01696	0.01156

Measure	20	100	200
sigma_Bias	-0.02546	-0.00517	-0.0038
sigma_AbsBias	0.02546	0.00517	0.0038
sigma_MSE	0.00668	0.0006	0.00027
sigma_RMSE	0.08171	0.02445	0.01635
Mean_AIC	-447.2855	-109.3621	-179.25605
Mean_BIC	-444.2983	-101.54659	-169.3611
Mean_HQIC	-446.70237	-106.19902	-175.25172

Table 7 shows that under low censoring, the **MG Akash regression model** becomes more accurate as sample size increases, with clear reductions in Bias, MSE, and RMSE for all parameters. Overall, larger samples lead to more precise and stable estimation, confirming strong model performance under low censoring.

4.6.2 Moderate Censoring (δ = 0.50)

Table 8. Bias MSE of parameter Model under Moderate Censoring

Measure	20	100	200
b0_Bias	0.08788	0.00246	0.00202
b0_AbsBias	0.08788	0.00246	0.00202
b0_MSE	1.57121	0.002	0.00075
b0_RMSE	1.25348	0.04467	0.02732
b1_Bias	0.02477	0.00719	0.00512
b1_AbsBias	0.02477	0.00719	0.00512
b1_MSE	0.09174	0.00085	0.00038
b1_RMSE	0.30289	0.02914	0.01937
sigma_Bias	-0.02703	0.00062	0.00086
sigma_AbsBias	0.02703	0.00062	0.00086
sigma_MSE	0.01161	0.00127	0.00052
sigma_RMSE	0.10777	0.0356	0.02282
Mean_AIC	-289.44643	-46.95843	-81.59935
Mean_BIC	-286.45923	-39.14291	-71.7044
Mean_HQIC	-288.86329	-43.79535	-77.59501

Table 8 shows that under moderate censoring, the **MG Akash Regression Model** improves significantly as the sample size increases. Bias, MSE, and RMSE decrease for all parameters (b_0, b_1, and σ), indicating more accurate and stable estimates. The improvement in information criteria also suggests better model fit, confirming that larger samples reduce the negative impact of censoring.

4.6.3 High Censoring (δ = 0.80)

Table 9. Bias ,MSE and RMSE Model under High Censoring

Measure	20	100	200
b0_Bias	0.48055	0.03505	0.0129
b0_AbsBias	0.48055	0.03505	0.0129
b0_MSE	34.81885	0.00943	0.00372
b0_RMSE	5.90075	0.09713	0.06096
b1_Bias	0.26487	0.03997	0.02458
b1_AbsBias	0.26487	0.03997	0.02458
b1_MSE	9.24805	0.00936	0.00292
b1_RMSE	3.04106	0.09676	0.05402

Measure	20	100	200
sigma_Bias	-0.0255	0.02381	0.02249
sigma_AbsBias	0.0255	0.02381	0.02249
sigma_MSE	0.05264	0.00799	0.00327
sigma_RMSE	0.22943	0.08941	0.05723
Mean_AIC	-123.09871	-1.88474	-2.96931
Mean_BIC	-120.11151	5.93077	6.92564
Mean_HQIC	-122.51558	1.27834	1.03503

Table 9 indicates that under high censoring, the **MG Akash Regression Model** performs poorly at small sample sizes, with large Bias, MSE, and RMSE values, especially for b_0 and b_1. However, as the sample size increases, all error measures decrease significantly, showing improved stability and precision. Although performance improves, estimation errors remain higher than in low and moderate censoring cases. Overall, larger samples partially reduce the adverse effect of high censoring, but do not fully eliminate it.

4.7 Residual analysis

Residual analysis is a fundamental diagnostic tool in regression that evaluates how well a model fits the data by examining the differences between observed and predicted values, known as residuals. By analyzing the pattern, spread, and distribution of these residuals, we can assess whether key assumptions—such as linearity, independence, constant variance (homoscedasticity), and normality—are satisfied. Ideally, residuals should be randomly scattered around zero with no clear pattern, indicating that the model has captured the underlying relationship effectively. Any systematic structure, unequal spread, or extreme values in the residuals may signal issues with the model, such as misspecification, nonlinearity, or the presence of outliers. We now present the residual analysis results for the first simulation case.

4.7.1. Residual analysis for simple MG Akash Regression Model case one

4.7.1.1. Pearson Residual analysis for simple MG Akash Regression Model case one

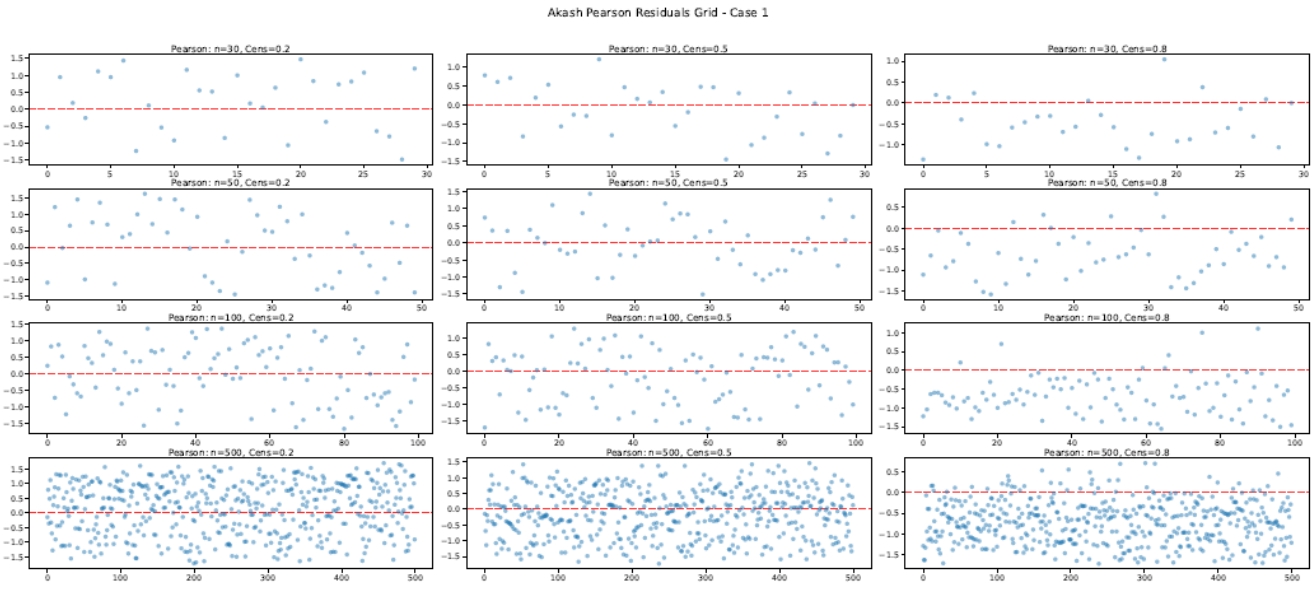


Figure 5. Pearson residuals for the MG Akash Regression Model (Case 1) across different sample sizes and censoring levels.

The Figure 5 shows the Pearson residuals for the Akash Regression Model (Case 1) across different sample sizes and censoring levels, where each point represents the standardized difference between observed and predicted values. The red dashed line at zero indicates a perfect fit, and the residuals are generally scattered around this line, suggesting that the model fits the data reasonably well. For smaller sample sizes (e.g., n=30), the residuals appear more dispersed and less structured, while larger samples (n=100, 500) show a more stable and evenly distributed pattern, indicating improved model reliability. As censoring increases, the spread of residuals becomes slightly more pronounced and may show mild asymmetry, reflecting reduced information. Overall, the plot suggests that the model performs better with larger sample sizes, while higher censoring slightly reduces the consistency of the fit.

4.7.1.2 Martginal Residual for simple MG Akash Regression Model case case one

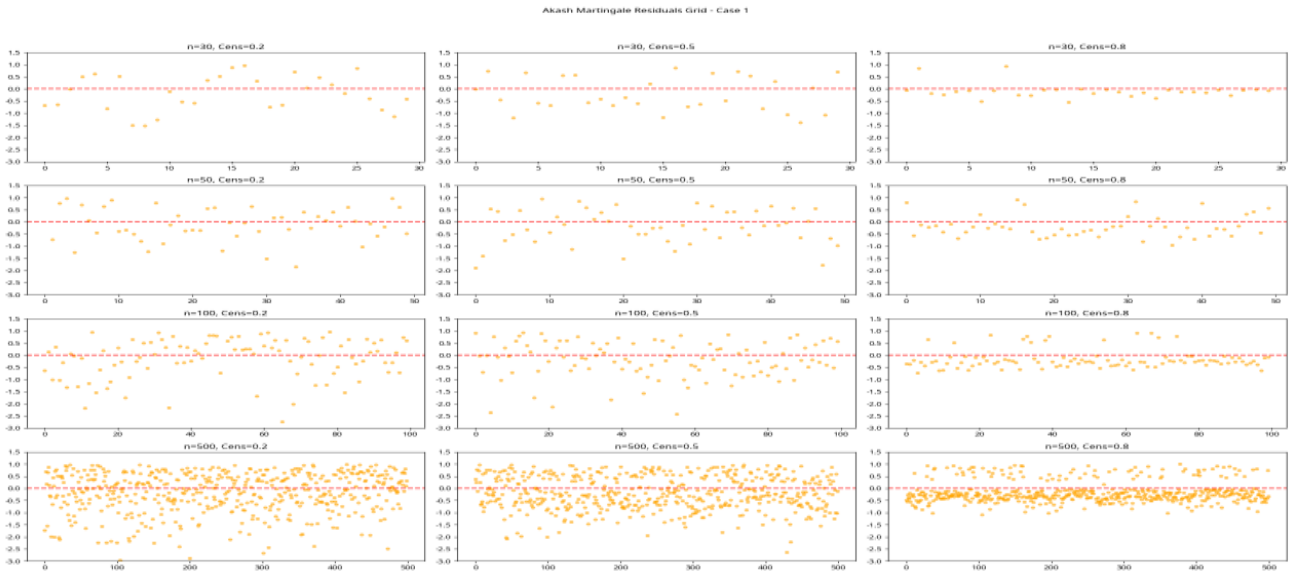


Figure 6. The Martingale residuals for the MG Akash Regression Model (Case 1) across different sample sizes and censoring levels.

This Figure 6 displays the Martingale residuals for the MG Akash Regression Model (Case 1) across different sample sizes and censoring levels, where each point represents the difference between observed and expected outcomes. The residuals are centered around the horizontal red dashed line at zero, indicating an overall reasonable model fit, although they show some asymmetry, which is typical for Martingale residuals. In smaller samples (n=30), the residuals appear more scattered and irregular, while larger samples (n=100, 500) show a more stable and consistent pattern. As censoring increases, the residuals tend to cluster more and display slightly increased variability, reflecting reduced information in the data. Overall, the figure suggests that the model fit improves with larger sample sizes, while higher censoring introduces more variability in the residual distribution.

4.7.1.3 Cooks distances for Simple MG Akash Regression Model case one

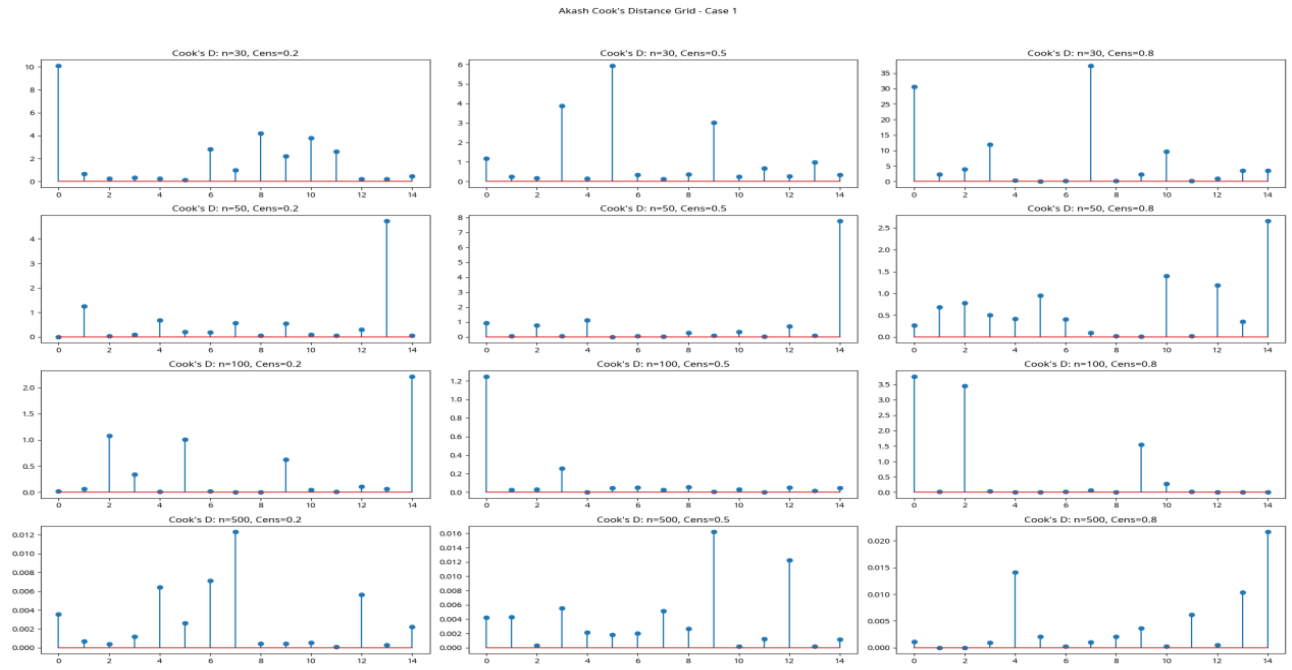


Figure 7. Cook's Distance diagnostics for MG Akash simple regression Case 1 across different sample sizes (n = and censoring levels.

The Figure 7 present **Cook's Distance diagnostics** for Case 1 across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8), where each vertical spike represents the influence of an individual observation on the fitted model. The red horizontal line indicates a reference threshold for identifying potentially influential points. For small sample sizes (especially n = 30), several observations show large Cook's Distance values, meaning a few data points have a strong impact on the model estimates. As the sample size increases, the magnitude of Cook's Distance values generally decreases and becomes more evenly distributed, indicating improved model stability and reduced sensitivity to individual observations. Higher censoring levels can slightly increase variability and lead to occasional influential points, but this effect diminishes with larger samples. Overall, the figure shows that the model becomes more robust and less affected by outliers as the sample size grow.

4.8 Likelihood Displacement for Simple MG akash Regression Model case one

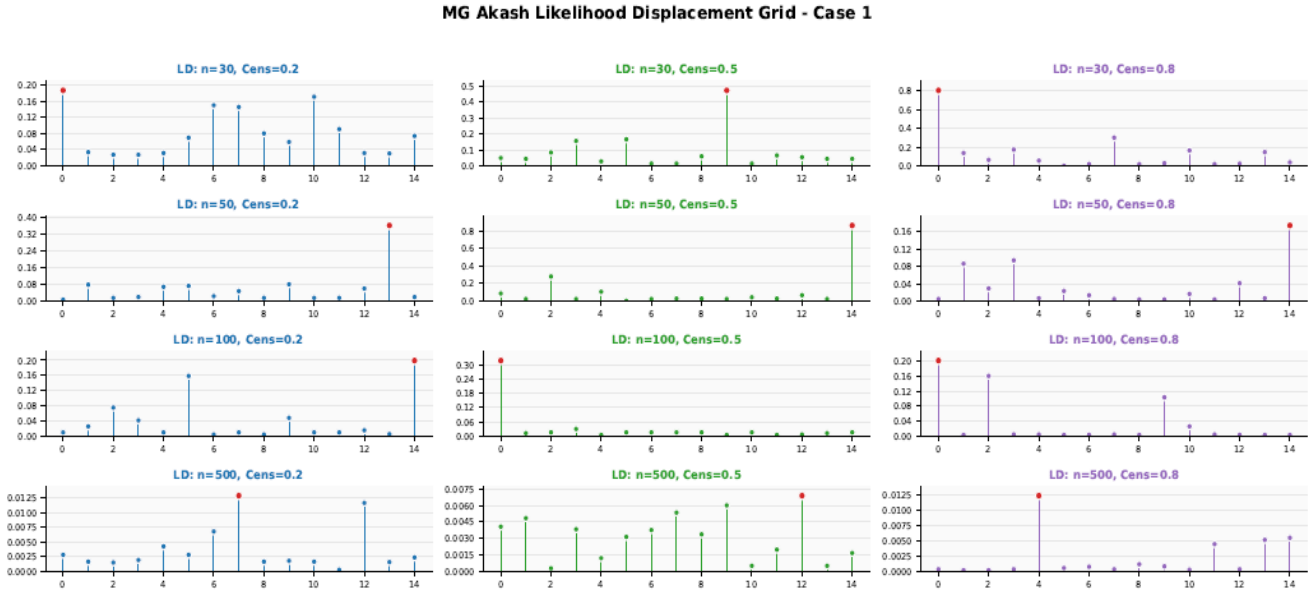


Figure 8. Likelihood Displacement (LD**)** for Case 1 under different sample sizes and censoring levels.

Figure 8 show the **Likelihood Displacement (LD)** for Case 1 under different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8), where each vertical spike represents how much the likelihood changes when a specific observation is perturbed or removed. Larger LD values indicate observations that have a stronger impact on the overall likelihood and thus are more influential. For small sample sizes (especially n = 30), several observations exhibit relatively high LD values, suggesting that the model is sensitive to certain data points. As the sample size increases, the LD values generally decrease and become more uniform, indicating improved stability and reduced influence of individual observations. Higher censoring levels tend to increase variability slightly, occasionally producing larger spikes, but this effect weakens with larger samples. Overall, the figure demonstrates that the model becomes more robust and less sensitive to influential observations as the sample size grows.

4.9 Residual analysis for simple MG Akash Regression Model (Case 2)

4.9.1 Pearson Residual analysis for simple MG Akash Regression Model case 2

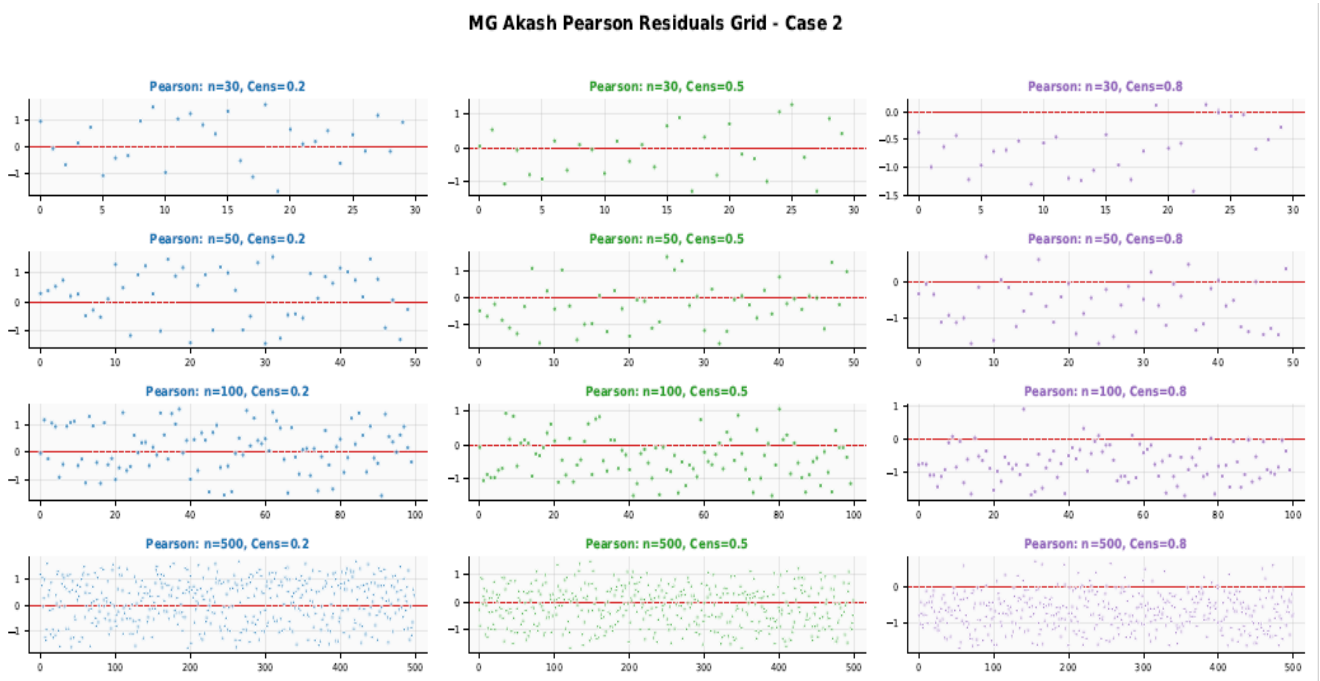


Figure 9. Pearson Residuals for Case 2 across different sample sizes and censoring levels.

These plots (Figure 9) show the **Pearson residuals** for Case 2 across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). Each point represents the residual of an observation, while the red dashed horizontal line at zero indicates perfect model fit. Overall, the residuals are randomly scattered around zero with no strong systematic pattern, suggesting that the model fits the data reasonably well. For smaller sample sizes, the residuals appear more dispersed and variable, while as the sample size increases, they become more densely clustered and stable. Higher censoring levels tend to shift more residuals below zero and slightly increase variability, indicating some loss of information. However, there are no clear trends or structures, which supports the assumption that the model is appropriate. Overall, the plots indicate a good fit, with improved reliability and consistency as the sample size grows.

4.9.2 Martginal Residual for simple MG Akash Regression Model case 2

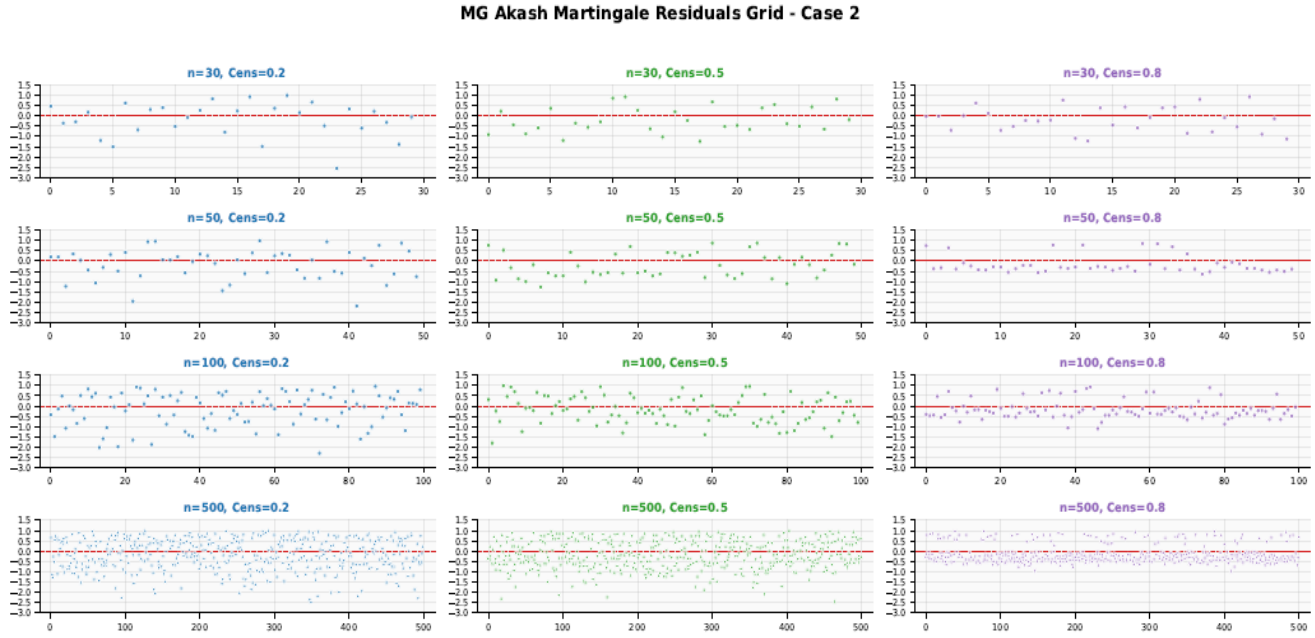


Figure 10. Martingale Residuals for Case 2 across different sample sizes and censoring levels.

These Figure 10 show the **Martingale residuals** for Case 2 across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). Each point represents the residual for an observation, with the red dashed horizontal line at zero serving as a reference. The residuals are generally scattered around zero without a strong systematic pattern, which indicates that the model provides an overall reasonable fit. For smaller sample sizes, the residuals appear more dispersed and include some extreme negative values, reflecting higher variability and potential sensitivity to individual observations. As the sample size increases, the residuals become more concentrated and stable, especially around zero, indicating improved model reliability. Higher censoring levels tend to compress the residuals slightly and introduce mild skewness (often toward negative values), reflecting reduced information in the data. Overall, the figure suggests that the model fit improves and becomes more stable with larger sample sizes despite the presence of censoring.

4.9.3 Cooks distances for Simple MG Akash Regression Model case 2

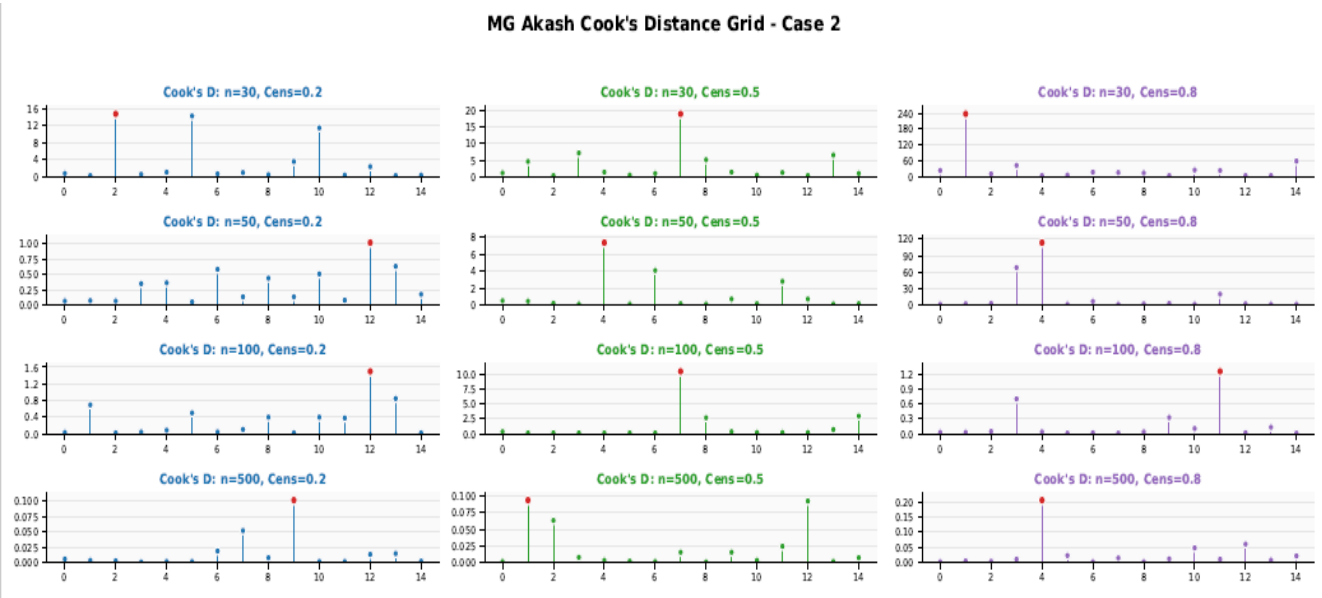


Figure 11. Cook's Distance values under Case 2 for different sample sizes and censoring levels**.**

This Figure 11 displays **Cook's Distance values under Case 2** for different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8), which are used to detect **influential observations** in the model. Each vertical spike represents how much a single observation affects the estimated parameters. A few **large spikes stand out**, especially for smaller sample sizes and higher censoring levels, indicating that certain observations have a strong influence on the model fit. As the sample size increases, the Cook's Distance values generally become **smaller and more stable**, meaning individual observations have less impact and the model becomes more robust. However, with higher censoring, some influential points still appear, suggesting that missing or incomplete data can increase sensitivity to certain observations. Overall, the plots show that the model is more stable with larger samples but can be affected by influential points, particularly under heavy censorin.

4.9.4 Likelihood Displacement for Simple MG akash Regression Model case 2

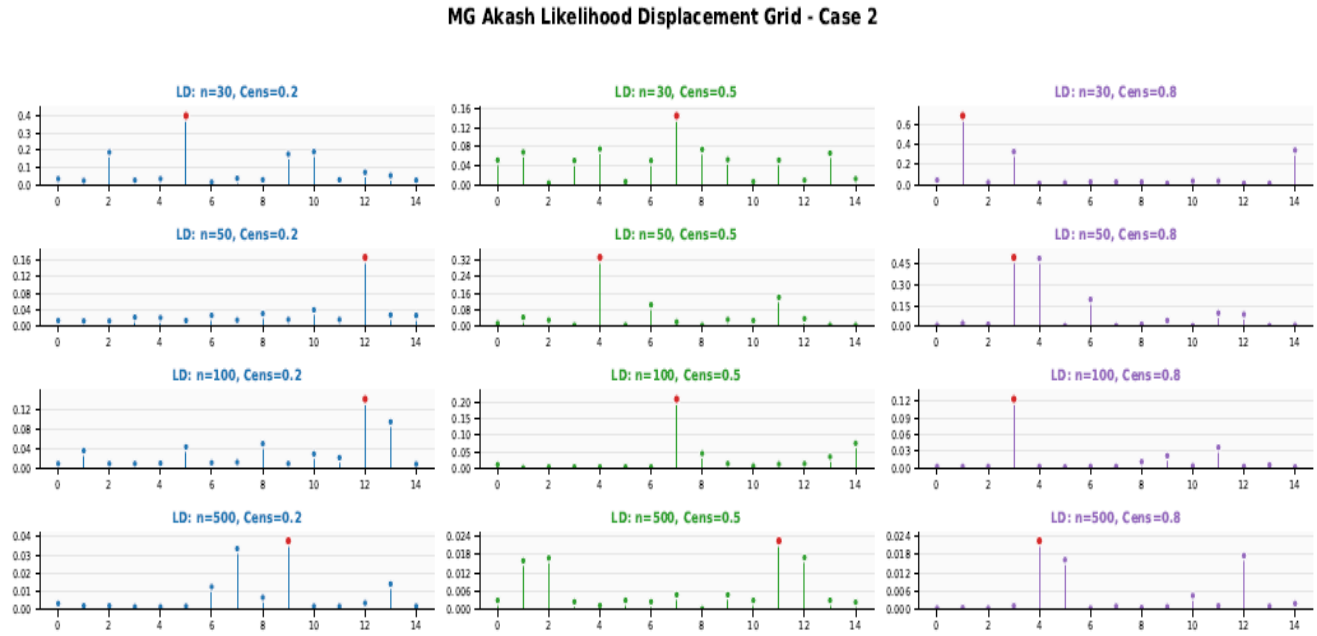


Figure 12. Likelihood Displacement (LD) values under Case 2 for different sample sizes and censoring levels.

The Figure 12 shows the **Likelihood Displacement (LD) values under Case 2** for different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8), which measure how much the overall likelihood changes when each observation is removed. Each spike represents the influence of a single observation on the model fit. We can see that a few observations produce **large LD values**, especially for smaller sample sizes and higher censoring levels, indicating strong influence on the likelihood and potential outliers. As the sample size increases, the LD values generally become **smaller and more evenly distributed**, showing that the model becomes more stable and less sensitive to individual data points. However, with heavier censoring, some influential observations still appear, suggesting that incomplete data increases sensitivity. Overall, the plots indicate that model stability improves with larger samples, while higher censoring can highlight influential observations.

4.10 Residual analysis for simple MG Akash Regression Model Case 3

4.10.1 Pearson Residual analysis for simple MG Akash Regression Model case 3

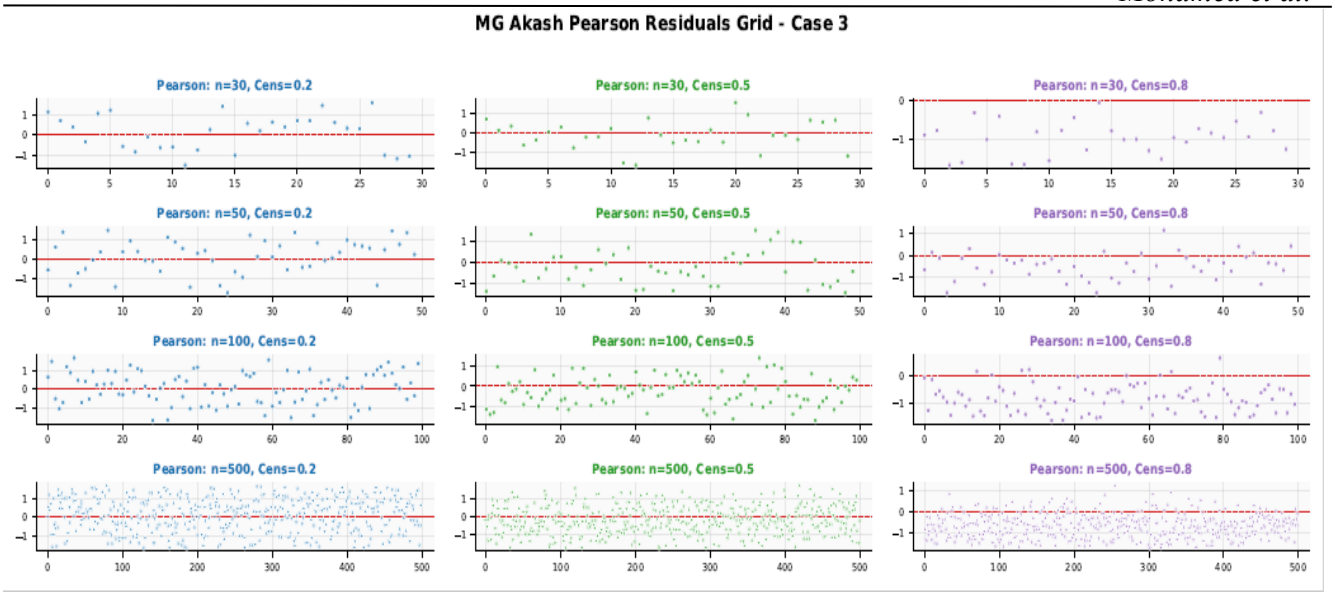


Figure 13. Pearson residuals for Case 3 across different sample sizes and censoring levels.

This Figure 13 shows the Pearson residuals for Case 3 across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). The residuals are plotted around the horizontal zero line (red dashed line), which represents a perfect model fit. Overall, the residuals appear randomly scattered without a clear pattern, indicating that the model fits the data reasonably well. As the sample size increases, the residuals become more densely and symmetrically distributed around zero, suggesting improved stability and reliability of the model. However, with higher censoring (especially 0.8), the residuals tend to be slightly more negative and somewhat less balanced, indicating a mild bias or loss of information due to heavy censoring. In general, the plots suggest that the model performs adequately, with better fit and consistency at larger sample sizes and lower censoring levels.

4.10. 2 Martginal Residual for simple MG Akash Regression Model case 3

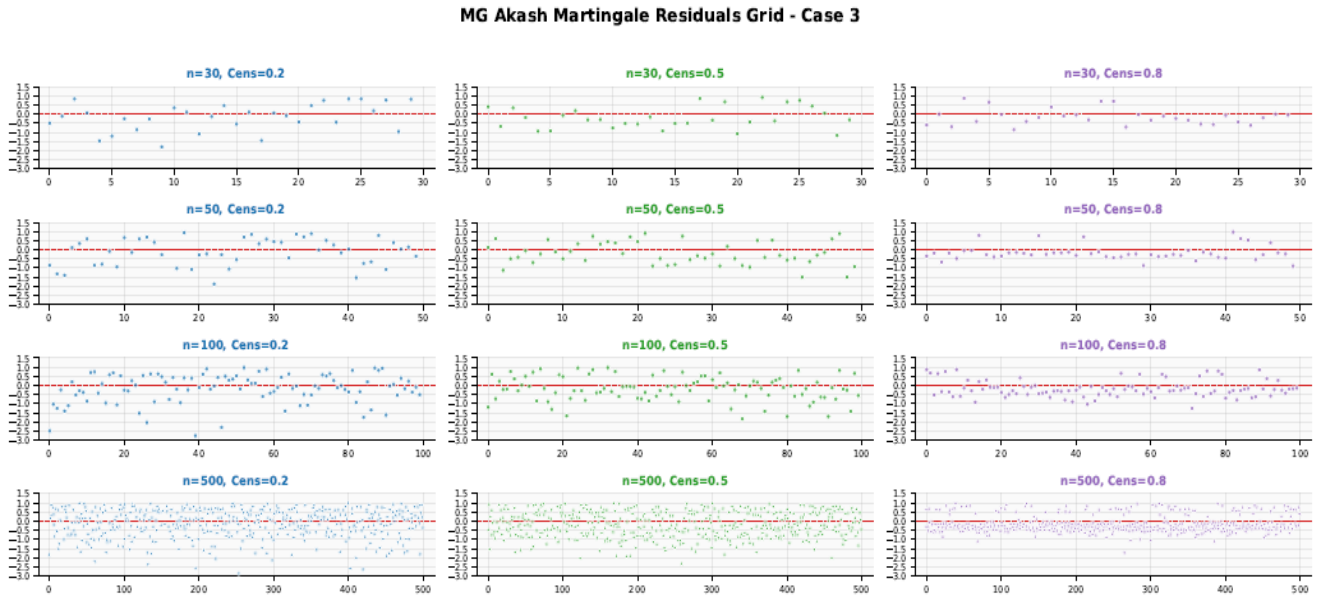


Figure 14. Martingale residuals grid (Case 3) for different sample sizes and censoring levels.

Figure 14 shows the **Martingale residuals grid (Case 3)** for different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). The residuals are plotted around the horizontal red line at zero. We observe that the points are not symmetrically distributed—there is a tendency for more negative values, which is a known property of Martingale residuals. As the sample size increases, the residuals become more stable and densely distributed, indicating improved reliability of the model. With higher censoring (0.8), the spread of the residuals shrinks and values cluster closer to zero, reflecting reduced variability. Overall, the absence of a clear pattern or systematic structure suggests that the model fits reasonably well, although the presence of some extreme negative values may indicate outliers or areas where the model could be improved.

4.11 Likelihood displacement (LD) for Simple MG Akash Regression Model case 3

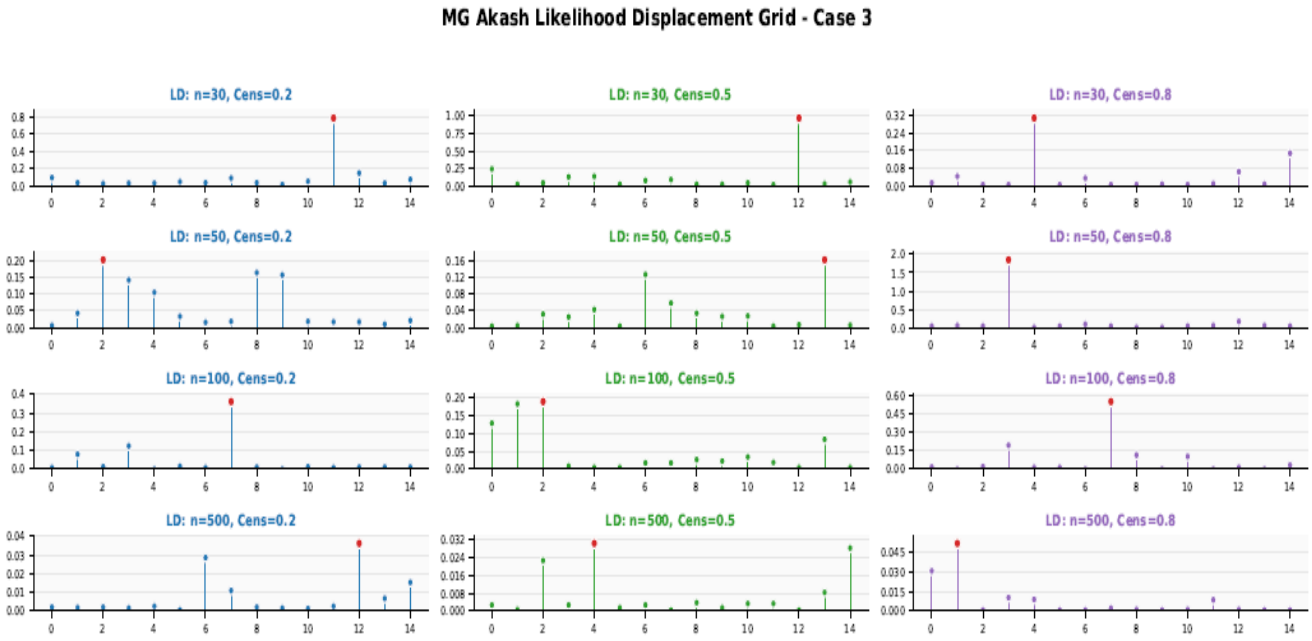


Figure 15. Likelihood displacement (LD) Case 3 across different sample sizes and censoring levels.

Figure 15 displays the **likelihood displacement (LD) grid for Case 3** across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). Each vertical spike represents how much the overall likelihood changes when a specific observation is perturbed or removed, so larger spikes indicate more influential observations. We can see that a few points stand out with notably higher LD values, meaning they have a strong influence on the model fit, while most observations have very small values and little impact. As the sample size increases, the influence of individual observations generally decreases and becomes more spread out, indicating greater model stability. However, at higher censoring levels (0.8), some observations still show relatively large influence, suggesting that censoring can amplify the impact of certain data points. Overall, the plot indicates that the model is fairly stable but may be sensitive to a small number of influential observations that should be examined more closely.

4.12 Cook's Distance for Simple MG Akash Regression Model case 3

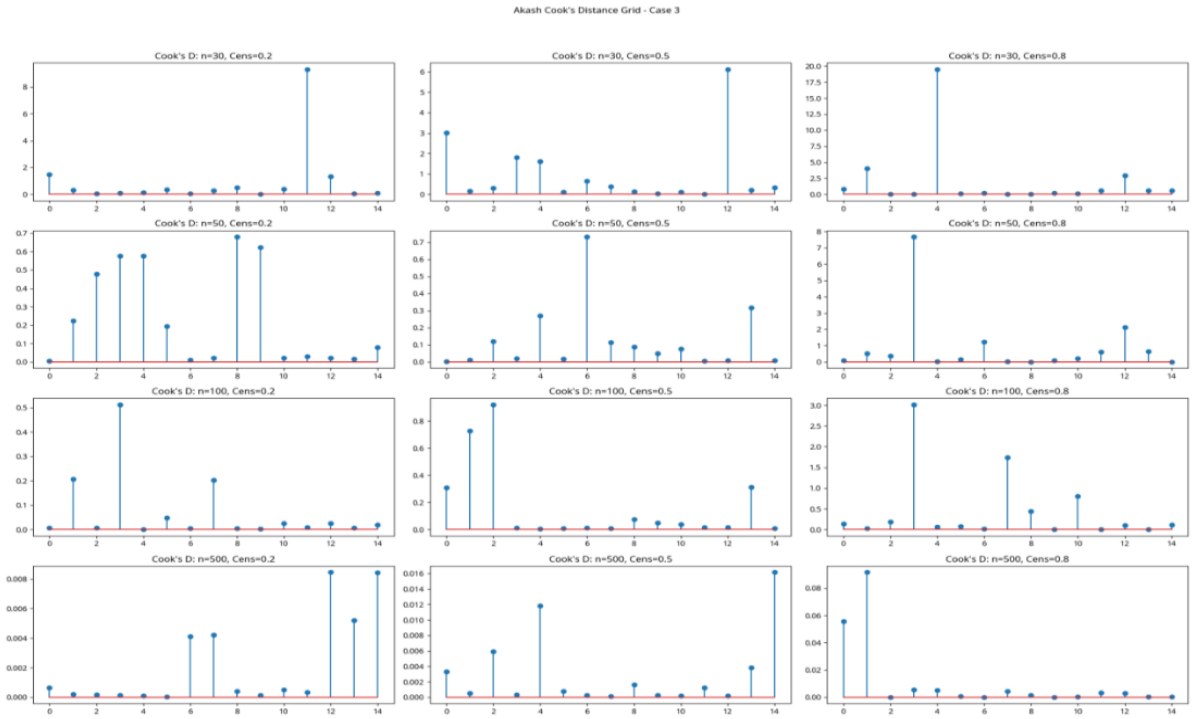


Figure 16. the Cook's Distance grid for Case 3 across different sample sizes and censoring levels.

This Figure 16 shows the **Cook's Distance grid for Case 3** across different sample sizes (n = 30, 50, 100, 500) and censoring levels (0.2, 0.5, 0.8). Each vertical spike represents the influence of an individual observation on the fitted model, where larger values indicate more influential points. We can see that a few observations have noticeably high Cook's Distance, meaning they strongly affect the model estimates, while most points have very small values and minimal impact. As the sample size increases, the overall influence of individual observations decreases and becomes more stable, indicating improved robustness of the model. However, at higher censoring levels (especially 0.8), some observations still show relatively large influence, suggesting that censoring can increase the sensitivity of the model to certain data points. Overall, the model appears generally stable, but a small number of influential observations should be carefully examined.

4.13 Real data

The following data represent the relationship between high blood pressure and the number of patients suffering from visual impairment, and the following presents some descriptive statistics.

Table 10. Destrective statistics high blood pressure and the number of patients

varaible	mean	Median	Q1	Q3
number of patients (y)	25	20.3	15	27
high blood pressure (x)	0.35	0.24	0.74	0.85

Table 11. Compare between MG Akash , gamma and Lindley Regression Model

Model	Bate note	Bate one	HQIC	AIC	BIC
MG Akash	0.5632	0.763	21.54	25.80	27.17
Gamma	0.8523	0.8321	23.56	35.84	34.93
Lindley	0.532	0.463	28.36	36.24	37.25

Tables 10 and 11 compares the performance of the **MG Akash, Gamma, and Lindley Regression Models** using parameter estimates and model selection criteria (AIC, BIC, and HQIC). The results show that the **MG Akash model** provides the best overall fit, as it has the lowest AIC (25.80), BIC (27.17), and relatively smaller HQIC compared to the other models. In contrast, the Gamma and Lindley models show higher values of AIC, BIC, and HQIC, indicating poorer model fit. Additionally, the parameter estimates of the mG Akash model are more stable compared to the other two models. Overall, the comparison confirms that the mG Akash regression model is more efficient and provides a better fit to the data than both Gamma and Lindley models.

5. Conclusions

The model parameters were estimated using the maximum likelihood approach. To evaluate its performance, the Minimum Guarantee Akash (MG Akash) Regression Model was compared with the Gamma and Lindley regression models using the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the likelihood ratio test. The results show that the proposed regression model outperforms the competing models in terms of goodness of fit and predictive performance. The mG Akash regression model also demonstrates strong flexibility and effectively captures heterogeneous variability in the data. These findings have important implications for economic modeling, such as risk prediction and reliability analysis, as well as for statistical theory, particularly in improving generalized regression frameworks and parameter estimation for non-negative and skewed data.

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Author Contributions

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