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ARTICLE

Neutrosophic ranked set sampling imputation methods for estimating imprecise population mean under missing data

Vishal Kumar and Anoop Kumar*

Department of Statistics, Central University of Haryana, Mahendergarh, India, 123031.
*Corresponding author. Email: anoop.asy@gmail.com

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Abstract

In survey sampling, estimating the population mean properly in the case of missing data is a difficult issue, while it becomes much more typical if the data are imprecise or uncertain. The conventional imputation methods frequently fail to handle the underlying uncertainty and imprecision in real-life datasets. To solve this issue, this paper adapts some fundamental neutrosophic imputations and proposes some efficient neutrosophic imputations under neutrosophic ranked set sampling (NeRSS) that successfully handles imprecise data while using neutrosophic logic principles. Through theoretical analysis and simulation study, we show that the proposed imputation methods yield more robust and trustworthy population mean estimates than the adapted ones, particularly in datasets with high missingness and imprecision. A real data insight is also presented in support of the theoretical and simulation findings.
Keywords: Neutrosophic imputation; Missing data; Population mean; Mean square error; Imprecise data.

1. Introduction

Missing data is a common problem in survey research that occurs due to equipment failures, data entry mistakes, and non-participation by respondents. This incompleteness can compromise the reliability of the conclusions, which might introduce bias and reduce statistical accuracy. However, appropriate imputation methods can lessen these impacts and improve the reliability and accuracy of results. Maintaining the integrity of survey research requires efficient handling of missing data. Rubin (1976) defined three fundamental concepts, namely, parameter distribution (PD), observed at random (OAR), and missing at random (MAR). Later, this idea was expanded upon by Heitjan and Basu (1996), who distinguished between MAR and missing completely at random (MCAR). Incorporating trustworthy auxiliary information is critical for improving the efficiency of the proposed imputation methods. Several imputation methods based on auxiliary information have been developed, with many assuming the MCAR mechanism. These methods seek to increase estimation accuracy and reduce bias by incorporating more relevant data.

Al-Omari (2011) computed the population mean via modified robust extreme ranked set sampling. Al-Omari *et al.* (2013) used several imputation procedures to estimate the population mean in the case of missing data using simple random sampling (SRS) and a known correlation coefficient. The optimal techniques for obtaining the population mean using imputation of missing data was developed by Bhushan and Pandey (2016). Prasad (2017) examined a number of ratio exponential type estimators with imputation for missing data in survey sampling. Sohail *et al.* (2019) studied the use of raw moments to impute missing data. Bhushan *et al.* (2023) used SRS to estimate the population mean in the presence of missing data. Sohail *et al.* (2023) conducted a comparative examination of several imputation methods. Rehman *et al.* (2024) developed and illustrated a new mean imputation process using abalone data. Yadav and Prasad (2024a) used a simulation study to develop a generalised class of factor type exponential imputation methods for population mean. Kumar *et al.* (2024) proposed some optimal imputation methods for population mean under correlated measurement errors.

Ranked set sampling (RSS) provides a cost-effective alternative to SRS, especially when ranking units are rather simple and affordable, but exact measurements are costly or time-consuming. McIntyre (1952) demonstrated the superiority of RSS over SRS in estimating the mean. Dell and Clutter (1972) provided a formal theoretical basis for using RSS in mean estimation and showed that it yields an unbiased estimate of the population mean. Several studies have highlighted the advantages of RSS over SRS for estimating the population mean. Stokes (1977) explicitly compared RSS with SRS and concluded that RSS substantially reduces variance of the estimator, especially in highly variable populations. Stokes and Sager (1988) successfully applied RSS in health research by using simplified ranking to efficiently estimate the biological marker concentrations. Chen *et al.* (2005) investigated RSS for efficient estimation of a population proportion. Zamanzadeh and Al-Omari (2016) introduced a new RSS procedures for estimating the population mean and variance. Rehman and Shabbir (2022) proposed an efficient class of estimators for the finite population mean under RSS in the presence of non-response. Zamanzadeh *et al.* (2024) studied the estimation of a decreasing mean residual life function based on RSS with applications to survival analysis. Kumar *et al.* (2024) introduced novel logarithmic imputation techniques in RSS utilizing multi-auxiliary information. Koopaei and Zamanzadeh (2025) explored nonparametric estimation of a biometric function under RSS in the presence of tie information.

Imprecise data refers to information that is inadequate, ambiguous, or variable, making it difficult to understand with absolute accuracy. Traditional analytical models, which rely on exact and well-defined values, may fail to interpret such data efficiently, thus leading to mistakes in decision-making. Managing imprecise data is especially important in industries such as healthcare, banking, and environmental monitoring, where choices are frequently made based on partial or ambiguous information. Advanced computational tools such as fuzzy logic, probabilistic models, and neutrosophic methods, contribute to these difficulties by allowing for more flexible and adaptable analysis, which leads to more robust and informed decision-making.

In survey sampling, the researches conducted till date have mostly focused on data that is precise, exact, and well characterised. When traditional methods are applied to such data, they yield a single, final estimate, which can sometimes result in an overestimation or underestimation of the underlying population parameter. This constraint is especially important when working with neutrosophic data, which is inherently ambiguous, indeterminate, and imprecise. In such instances, traditional estimation procedures frequently fail, emphasizing the necessity for neutrosophic statistics. Furthermore, data collection can be expensive, especially when working with imprecise or partial information. Estimating an unknown population parameter under imprecise conditions using classical estimation methods not only expensive but also risky. Literature of survey sampling contains very limited studies for imprecise mean estimation under different sampling schemes. Tahir *et al.* (2021) suggested a neutrosophic ratio-based population mean estimation using SRS. Yadav and Prasad (2024b) introduced neutrosophic estimators for estimating the population mean in survey sampling. Alomair and Shahzad (2023) introduced neutrosophic mean estimation for sensitive and non-sensitive variables using robust Hartley-Ross type estimators. For mean estimate, Alqudah *et al.* (2024) introduced the neutrosophic robust ratio-type estimator of population mean. Masood *et al.* (2024) proposed a robust estimation method to compute the neutrosophic finite median of the auxiliary variable. Singh and Gupta (2024) explored the combined use of two auxiliary variables to improve the finite population mean estimation under a neutrosophic framework. Al-Marzouki and Ahmad (2025) proposed an enhanced class of neutrosophic estimators with an application to Islamabad Stock Exchange. Alomair and Ahmad (2025) introduced a comprehensive regression-cum-exponential type estimator for neutrosophic data. Moreover, Shahzad *et al.* (2025) developed neutrosophic extensions of Horvitz-Thompson type estimators. Kumar and Kumar (2025a) introduced neutrosophic imputation methods for estimating the population mean of climate data under SRS. Kumar and Kumar (2025b) introduced a simulation based evaluation of neutrosophic exponential imputations of population mean using NeRSS. Priya and Kumar (2025) proposed the robust neutrosophic exponential estimators under uncertainty. Kumar and Priya (2026) introduced a novel class of neutrosophic estimators for mean estimation under indeterminacy.

Numerous imputation methods based on the MCAR mechanism have been proposed in the literature to address missing values within certain datasets under the RSS framework. These methods are often built to deal with precisely defined missing values and frequently overlook the inherent ambiguity and uncertainty that come with missing information under RSS. When uncertainty is added, missing data becomes much more complex, making traditional imputation techniques insufficient for precise population parameter estimation. To overcome these issues, we develop some efficient neutrosophic imputation approaches that can accommodate such complexities and improve the population mean estimation.

1.1 Notations

The RSS approach has been carefully examined and found to be a more efficient alternative to the SRS methodology in cases when precise measurements are impossible, expensive, or time-consuming. This is due to RSS's use of supplemental population information in the form of rankings, which results in more accurate estimates with fewer measurements. However, when dealing with inaccurate, inconsistent, or imprecise data, as is usual in a neutrosophic situation, measurement difficulties become much more obvious. The statistical literature provides various types of neutrosophic data including quantitative neutrosophic data based on a number inside an undetermined interval $[p,\ q]$, where 'p' and 'q' represent the lower and upper values of the neutrosophic data. The unknown interval $[p,\ q]$ based on neutrosophic numbers can be represented in different ways. The neutrosophic interval values are computed as $Z_{rssN} = Z_{rssL} + Z_{rssU} I_{rssN}$, where $I_{rssN} \in [I_{rssL},\ I_{rssU}]$, N in the subscript stands for neutrosophic number. This demonstrates that the notations utilized for neutrosophic data are in an interval form $Z_{rssN} \in [p,\ q]$.

The NeRSS approach utilizes bivariate random samples of size $m_N \in [m_L, m_U]$ units from a population of size N. Within each sample, the auxiliary variable $x_N \in [x_L,\ x_U]$ is ranked against the study variable $y_N \in [y_L,\ y_U]$. The ranking of neutrosophic numbers may be simply accomplished using the method introduced by Smarandache (2005).

In the NeRSS, select the smallest unit from the first set and use it as the starting measuring unit, discarding the other units in the set. Similarly, from the second set, select the second smallest unit and consider it the second measuring unit, discarding the other units in the set. To continue, choose the m_N^{th} smallest unit from the m_N^{th} collection and utilize it as the m_N^{th} measurement unit, discarding the remaining units. This procedure concludes a cycle. Repeating this cycle k times results in a neutrosophic ranked set sample of size $n_N = m_N k \in [n_L,\ n_U]$. The extraction of NeRSS data requires a total of $m_N^2 k$ units, although only $n_N = m_N k \in [n_L, n_U]$ units are necessary for calculation. Let $X_{j(i)N} \in [X_{j(i)L}, X_{j(i)U}]$ and $Y_{j[i]N} \in [Y_{j[i]L},\ Y_{j[i]U}]$, $j = 1, 2, ..., k$; $i = 1, 2, ..., m_N$ denote a pair of neutrosophic bivariate quantified sets of the i^{th} units in the j^{th} cycle. The symbols '[]' and '()' in the subscripts of variables y_N and x_N represent the imperfect and perfect rankings of units, respectively.

We consider a neutrosophic random sample of size n_N, selected from a finite population of N units. Let $x_N(i) \in [x_L,\ x_U]$ and $y_N(i) \in [y_L,\ y_U]$ express the information on the neutrosophic auxiliary variable and neutrosophic study variable observed in the i^{th} sample. From the retrieved n_N units, r_N represents the number of neutrosophic responsive units. Let P_N be the probability of responding units. The R_N is the set of neutrosophic responding units, while $\bar{R}_N$ is the set of non-responding units. The value $y_{N(i)}$ is obtained for each unit $i \in R_N$. To complete the dataset, however, the values for the units $i \in \bar{R}_N$ must be imputed because they are missing. If the imputation is done using additional neutrosophic auxiliary information $x_{N(i)}$, then the data $x_{s_N} = x_{i_N}$ are available because the value of x_N for i^{th} unit is known and positive for all $i \in s_N$. It is assumed that $\bar{Y}_N \in [\bar{Y}_L,\ \bar{Y}_U]$ and $\bar{X}_N \in [\bar{X}_L,\ \bar{X}_U]$ are the population means that correspond to the sample means of the neutrosophic primary variable y_N as well as the neutrosophic auxiliary variable x_N, $\bar{y}_N \in [\bar{y}_L,\ \bar{y}_U]$ and $\bar{x}_N \in [\bar{x}_L,\ \bar{x}_U]$, respectively. Suppose that $C_{y_N} \in [C_{y_L},\ C_{y_U}]$ and $C_{x_N} \in [C_{x_L},\ C_{x_U}]$ for the neutrosophic variables y_N and x_N, are their respective neutrosophic coefficients of variation. With regard to neutrosophic variables y_N and x_N, the correlation coefficient is ρ_{xy_N}.

To determine the mean square error (MSE) $MSE_N \in [MSE_L,\ MSE_U]$ and $Bias_N \in [Bias_L,\ Bias_U]$ of the resulting neutrosophic estimators, the neutrosophic error terms $\epsilon_{0N} \in [\epsilon_{0L},\ \epsilon_{0U}]$, $\epsilon_{1N} \in [\epsilon_{1L},\ \epsilon_{1U}]$, and $\epsilon_{2N} \in [\epsilon_{2L},\ \epsilon_{2U}]$, together with the following expectations:

$$\left.\begin{aligned}
\epsilon_{0N} &= \frac{\bar{y}_{r_N,rss} - \bar{Y}_N}{\bar{Y}_N}, \quad \epsilon_{1N} = \frac{\bar{x}_{r_N,rss} - \bar{X}_N}{\bar{X}_N}, \quad \epsilon_{2N} = \frac{\bar{x}_{n_N,rss} - \bar{X}_N}{\bar{X}_N},\\
E(\epsilon_{0N}) &= E(\epsilon_{1N}) = E(\epsilon_{2N}) = 0,\\
E(\epsilon_{0N}^2) &= \frac{C_{y_N}^2}{m_N k P_N} - \frac{1}{m_N^2 k P_N}\sum_{i=1}^{m_N}\frac{\tau_{y_{N[i]}}^2}{\bar{Y}_N^2} = V_0^*,\\
E(\epsilon_{1N}^2) &= \frac{C_{x_N}^2}{m_N k P_N} - \frac{1}{m_N^2 k P_N}\sum_{i=1}^{m_N}\frac{\tau_{x_{N(i)}}^2}{\bar{X}_N^2} = V_1^*,\\
E(\epsilon_{2N}^2) &= \frac{C_{x_N}^2}{m_N k} - \frac{1}{m_N^2 k}\sum_{i=1}^{m_N}\frac{\tau_{x_{N(i)}}^2}{\bar{X}_N^2} = V_1,\\
E(\epsilon_{0N}\epsilon_{1N}) &= \frac{\rho_{xy_N} C_{x_N} C_{y_N}}{m_N k P_N} - \frac{1}{m_N^2 k P_N}\sum_{i=1}^{m_N}\frac{\tau_{xy_{N[i]}}}{\bar{X}_N \bar{Y}_N} = V_{01}^*,\\
E(\epsilon_{0N}\epsilon_{2N}) &= \frac{\rho_{xy_N} C_{x_N} C_{y_N}}{m_N k} - \frac{1}{m_N^2 k}\sum_{i=1}^{m_N}\frac{\tau_{xy_{N[i]}}}{\bar{X}_N \bar{Y}_N} = V_{01},\\
E(\epsilon_{1N}\epsilon_{2N}) &= \frac{C_{x_N}^2}{m_N k} - \frac{1}{m_N^2 k}\sum_{i=1}^{m_N}\frac{\tau_{x_{N(i)}}^2}{\bar{X}_N^2} = V_1,
\end{aligned}\right\} \tag{1.1}$$

where $\tau_{y_{N[i]}} = (\mu_{y_{N[i]}} - \bar{Y}_N)$, $\tau_{x_{N(i)}} = (\mu_{x_{N(i)}} - \bar{X}_N)$, $\tau_{xy_{[i]}} = (\mu_{x_{N(i)}} - \bar{X}_N)(\mu_{y_{N[i]}} - \bar{Y}_N)$, $C_{y_N} = S_{y_N}/\bar{Y}_N$, $C_{x_N} = S_{x_N}/\bar{X}_N$, $\mu_{y_{N[i]}} = E(Y_{N[i]})$, and $\mu_{x_{N(i)}} = E(X_{N(i)})$.

This article is further divided into different sections. Some well-known neutrosophic imputation techniques and the related neutrosophic estimators are modified in Section 2 to close the gap in the literature review. The proposed efficient neutrosophic imputation methods, the resulting neutrosophic estimators, and their associated properties are provided in Section 3. In Section 4, the adapted estimators and the proposed resulting estimators are compared. In Section 5, a comprehensive simulation investigation is carried out with symmetric and asymmetric hypothetical neutrosophic populations. Section 6 analyzes the resultant neutrosophic estimators that have been modified and suggested using actual data. The study's conclusion is written in Section 7.

2. Adapted neutrosophic imputation methods

There are currently no neutrosophic imputation methods in the literature for handling missing data in NeRSS. To fill this gap and evaluate the efficiency of the suggested neutrosophic imputation methods and consequent estimators, we adapt some fundamental neutrosophic imputation methods and consequent neutrosophic estimators while taking into account their specific properties.

2.1 Neutrosophic mean imputation method

The conventional mean imputation fails with imprecise datasets because it is dependent on certainty assumptions. The neutrosophic mean imputation method provides a more comprehensive and adaptive method for handling missing values by taking into consideration degrees of truth, indeterminacy, and untruth. In accordance with Lee *et al.* (1994), the neutrosophic mean imputation method is as follows when a sample unit 'i' is absent and needs to be imputed:

$$y_{.i_m} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \bar{y}_{r_N,rss} & \text{if } i \in \bar{R}_N \end{cases}$$

The resulting neutrosophic mean estimate is provided as

$$T_m = \bar{y}_{r_N,rss}.$$

The variance of the resultant neutrosophic mean estimator $T_m = \bar{y}_{r_N,rss}$ is provided by

$$V(T_m) = \bar{Y}_N^2 V_0^*.$$

When additional neutrosophic auxiliary data are available, the neutrosophic imputation methods are divided into three schemes, which are detailed below.
Scheme I: When $\bar{X}_N$ is available, consider $\bar{x}_{n_N,rss}$.
Scheme II: When $\bar{X}_N$ is available, consider $\bar{x}_{r_N,rss}$.
Scheme III: When $\bar{X}_N$ is not available, consider $\bar{x}_{n_N,rss}$ and $\bar{x}_{r_N,rss}$.

2.2 Neutrosophic ratio imputation methods

According to Cochran (1977), the ratio estimator perform better when study and auxiliary variables are positively correlated. Following Cochran (1977), when neutrosophic study and neutrosophic auxiliary variables are positively correlated, then the neutrosophic ratio imputation methods produce efficient results which are described under schemes I–III as
Scheme I

$$y_{.i_{r_1}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left[n_N \bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{n_N,rss}}\right) - r_N \bar{y}_{r_N,rss}\right] & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{.i_{r_2}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left[n_N \bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{r_N,rss}}\right) - r_N \bar{y}_{r_N,rss}\right] & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{.i_{r_3}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left[n_N \bar{y}_{r_N,rss}\left(\frac{\bar{x}_{n_N,rss}}{\bar{x}_{r_N,rss}}\right) - r_N \bar{y}_{r_N,rss}\right] & \text{if } i \in \bar{R}_N \end{cases}$$

The resulting estimators of neutrosophic ratios under schemes I–III are provided by

$$T_{r_1} = \bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{n_N,rss}}\right),$$
$$T_{r_2} = \bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{r_N,rss}}\right),$$
$$\text{and } \; T_{r_3} = \bar{y}_{r_N,rss}\left(\frac{\bar{x}_{n_N,rss}}{\bar{x}_{r_N,rss}}\right).$$

The approximate MSE of the neutrosophic ratio estimators is given by

$$MSE(T_{r_1}) \approx \bar{Y}_N^2(V_0 + V_1 - 2V_{01}),$$
$$MSE(T_{r_2}) \approx \bar{Y}_N^2(V_0^* + V_1^* - 2V_{01}^*),$$
$$\text{and } \; MSE(T_{r_3}) \approx \bar{Y}_N^2\left\{V_0^* + (V_1^* - V_1) - 2(V_{01}^* - V_{01})\right\}.$$

2.3 Neutrosophic regression imputation methods

When neutrosophic study and neutrosophic auxiliary variables are positively correlated, and the regression line goes from the origin, then following Cochran (1977) and Diana and Perri (2010), the traditional neutrosophic regression imputation methods produce efficient results which are described under schemes I–III as

Scheme I

$$y_{.i_{lr_1}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \bar{y}_{r_N,rss} + \frac{n_N \beta_1}{n_N - r_N}(\bar{X}_N - \bar{x}_{n_N,rss}) & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{.i_{lr_2}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \bar{y}_{r_N,rss} + \frac{n_N \beta_2}{n_N - r_N}(\bar{X}_N - \bar{x}_{r_N,rss}) & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{.i_{lr_3}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \bar{y}_{r_N,rss} + \frac{n_N \beta_3}{n_N - r_N}(\bar{x}_{n_N,rss} - \bar{x}_{r_N,rss}) & \text{if } i \in \bar{R}_N \end{cases}$$

The resulting estimators for neutrosophic regression are provided as

$$T_{lr_1} = \bar{y}_{r_N,rss} + \beta_1(\bar{X}_N - \bar{x}_{n_N,rss}),$$
$$T_{lr_2} = \bar{y}_{r_N,rss} + \beta_2(\bar{X}_N - \bar{x}_{r_N,rss}),$$
$$\text{and } \; T_{lr_3} = \bar{y}_{r_N,rss} + \beta_3(\bar{x}_{n_N,rss} - \bar{x}_{r_N,rss}).$$

The regression coefficients are $\beta_i,\ i=1,2,3$. These scalars' ideal values are provided by

$$\beta_{1(opt)} = \frac{RV_{01}}{V_1},$$
$$\beta_{2(opt)} = \frac{RV_{01}^*}{V_1^*},$$
$$\text{and}\quad \beta_{3(opt)} = \frac{R(V_{01}^*-V_{01})}{(V_1^*-V_1)}.$$

where $R=\bar{Y}_N/\bar{X}_N$ is the population ratio.
The minimum MSE of the resultant neutrosophic regression estimators is given by

$$min.MSE(T_{lr_1}) = \bar{Y}_N^2\left(V_0^*-\frac{V_{01}^2}{V_1}\right),$$
$$min.MSE(T_{lr_2}) = \bar{Y}_N^2\left(V_0^*-\frac{{V_{01}^*}^2}{V_1^*}\right),$$
$$\text{and}\quad min.MSE(T_{lr_3}) = \bar{Y}_N^2\left\{V_0^*-\frac{(V_{01}^*-V_{01})^2}{(V_1^*-V_1)}\right\}.$$

2.4 Bhushan *et al.* (2023) type neutrosophic imputation methods

Searls (1964) suggested pre-multiplying a scalar to the estimator improve its efficiency. Following Searls (1964) and Bhushan *et al.* (2023), we adapt some Searls type neutrosophic regression and ratio estimators under schemes I–III as
Scheme I

$$y_{.i_{b_1}} = \begin{cases} \zeta_1 y_{i_N} & \text{if } i\in R_N \\ \zeta_1\bar{y}_{r_N,rss}+\frac{n_N\theta_1}{n_N-r_N}(\bar{x}_{n_N,rss}-\bar{X}_N) & \text{if } i\in\bar{R}_N \end{cases}$$

$$y_{.i_{b_4}} = \begin{cases} y_{i_N} & \text{if } i\in R_N \\ \frac{1}{n_N-r_N}\left(n_N\zeta_4\bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{n_N,rss}}\right)^{\theta_4}-r_N\bar{y}_{r_N,rss}\right) & \text{if } i\in\bar{R}_N \end{cases}$$

Scheme II

$$y_{.i_{b_2}} = \begin{cases} \zeta_2 y_{i_N} & \text{if } i\in R_N \\ \zeta_2\bar{y}_{r_N,rss}+\frac{n_N\theta_2}{n_N-r_N}(\bar{x}_{r_N,rss}-\bar{X}_N) & \text{if } i\in\bar{R}_N \end{cases}$$

$$y_{.i_{b_5}} = \begin{cases} y_{i_N} & \text{if } i\in R_N \\ \frac{1}{n_N-r_N}\left(n_N\zeta_5\bar{y}_{r_N,rss}\left(\frac{\bar{X}_N}{\bar{x}_{r_N,rss}}\right)^{\theta_5}-r_N\bar{y}_{r_N,rss}\right) & \text{if } i\in\bar{R}_N \end{cases}$$

Scheme III

$$y_{.i_{b_3}} = \begin{cases} \zeta_3 y_{i_N} & \text{if } i\in R_N \\ \zeta_3\bar{y}_{r_N,rss}+\frac{n_N\theta_3}{n_N-r_N}(\bar{x}_{r_N,rss}-\bar{x}_{n_N,rss}) & \text{if } i\in\bar{R}_N \end{cases}$$

$$y_{.i_{b_6}} = \begin{cases} y_{i_N} & \text{if } i\in R_N \\ \frac{1}{n_N-r_N}\left(n_N\zeta_6\bar{y}_{r_N,rss}\left(\frac{\bar{x}_{n_N,rss}}{\bar{x}_{r_N,rss}}\right)^{\theta_6}-r_N\bar{y}_{r_N,rss}\right) & \text{if } i\in\bar{R}_N \end{cases}$$

Under the above schemes, the resultant neutrosophic estimators are given as

$$
\begin{aligned}
T_{b_1} &= \zeta_1 \bar{y}_{r_N,rss} + \theta_1(\bar{x}_{n_N,rss} - \bar{X}_N),\\
T_{b_2} &= \zeta_2 \bar{y}_{r_N,rss} + \theta_2(\bar{x}_{r_N,rss} - \bar{X}_N),\\
T_{b_3} &= \zeta_3 \bar{y}_{r_N,rss} + \theta_3(\bar{x}_{r_N,rss} - \bar{x}_{n_N,rss}),\\
T_{b_4} &= \zeta_4 \bar{y}_{r_N,rss} \left(\frac{\bar{X}_N}{\bar{x}_{n_N,rss}}\right)^{\theta_4},\\
T_{b_5} &= \zeta_5 \bar{y}_{r_N,rss} \left(\frac{\bar{X}_N}{\bar{x}_{r_N,rss}}\right)^{\theta_5},\\
\text{and}\quad T_{b_6} &= \zeta_6 \bar{y}_{r_N,rss} \left(\frac{\bar{x}_{n_N,rss}}{\bar{x}_{r_N,rss}}\right)^{\theta_6}.
\end{aligned}
$$

where ζ_i and θ_i, $i = 1,2,...,6$ are suitably chosen scalars.
The minimum MSE of the resultant neutrosophic estimators T_{b_i}, $i = 1,2,...,6$ is given by

$$
min.MSE(T_{b_i}) = \bar{Y}_N^2 \left(1 - \frac{H_i^2}{G_i}\right).
$$

where

$$
\begin{aligned}
H_1 &= H_2 = H_3 = 1,\\
G_1 &= 1 + V_0^* - \frac{V_{01}^2}{V_1},\\
G_2 &= 1 + V_0^* - \frac{{V_{01}^*}^2}{V_1^*},\\
G_3 &= 1 + V_0^* - \frac{(V_{01}^* - V_{01})^2}{(V_1^* - V_1)},\\
G_4 &= 1 + V_0^* + \theta_4(2\theta_4 + 1)V_1 - 4\theta_4 V_{01},\\
H_4 &= 1 - \theta_4 V_{01} + \left\{\frac{\theta_4(\theta_4 + 1)}{2}\right\} V_1,\\
G_5 &= 1 + V_0^* + \theta_5(2\theta_5 + 1)V_1^* - 4\theta_5 V_{01}^*,\\
H_5 &= 1 - \theta_5 V_{01}^* + \left\{\frac{\theta_5(\theta_5 + 1)}{2}\right\} V_1^*,\\
G_6 &= 1 + V_0^* + \theta_6(2\theta_6 + 1)(V_1^* - V_1) - 4\theta_6(V_1^* - V_{01}),\\
H_6 &= 1 - \theta_6(V_{01}^* - V_{01}) + \left\{\frac{\theta_6(\theta_6 + 1)}{2}\right\}(V_1^* - V_1).
\end{aligned}
$$

3. Proposed neutrosophic imputation methods

In survey sampling, dealing with the missingness in uncertain data is a big issue for survey practitioners. To overcome this issue, we propose some efficient neutrosophic imputation methods for the population mean using SRS. These imputation methods are provided under schemes I–III as follows:

Scheme I

$$y_{.i_{v_1}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{c} \left[\begin{array}{l} \alpha_1 \bar{y}_{r_N,rss} + \\ \gamma_1 \bar{y}_{r_N,rss} \left\{ \dfrac{\bar{X}_N}{\theta_N \bar{x}_{n_N,rss} + (1-\theta_N)\bar{X}_N} \right\}^g \end{array} \right] \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{n_N,rss}}{\bar{X}_N} \right) \right\}^{\phi_1} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

$$y_{.i_{v_4}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{l} \{\alpha_4 \bar{y}_{r_N,rss} + \gamma_4(\bar{x}_{n_N,rss} - \bar{X}_N)\} \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{n_N,rss}}{\bar{X}_N} \right) \right\}^{\phi_4} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{.i_{v_2}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{c} \left[\begin{array}{l} \alpha_2 \bar{y}_{r_N,rss} + \\ \gamma_2 \bar{y}_{r_N,rss} \left\{ \dfrac{\bar{X}_N}{\theta_N \bar{x}_{r_N,rss} + (1-\theta_N)\bar{X}_N} \right\}^g \end{array} \right] \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{r_N,rss}}{\bar{X}_N} \right) \right\}^{\phi_{2N}} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

$$y_{.i_{v_5}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{l} \{\alpha_5 \bar{y}_{r_N,rss} + \gamma_5(\bar{x}_{r_N,rss} - \bar{X}_N)\} \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{r_N,rss}}{\bar{X}_N} \right) \right\}^{\phi_5} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{.i_{v_3}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{c} \left[\begin{array}{l} \alpha_3 \bar{y}_{r_N,rss} + \\ \gamma_3 \bar{y}_{r_N,rss} \left\{ \dfrac{\bar{x}_{n_N,rss}}{\theta_N \bar{x}_{r_N,rss} + (1-\theta_N)\bar{x}_{n_N,rss}} \right\}^g \end{array} \right] \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{r_N,rss}}{\bar{x}_{n_N,rss}} \right) \right\}^{\phi_3} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

$$y_{.i_{v_6}} = \begin{cases} y_{i_N} & \text{if } i \in R_N \\ \frac{1}{n_N - r_N}\left(n_N \left[\begin{array}{l} \{\alpha_6 \bar{y}_{r_N,rss} + \gamma_6(\bar{x}_{r_N,rss} - \bar{x}_{n_N,rss})\} \\ \times \left\{ 1 + \log\left(\dfrac{\bar{x}_{r_N,rss}}{\bar{x}_{n_N,rss}} \right) \right\}^{\phi_6} \end{array} \right] - r_N \bar{y}_{r_N,rss} \right) & \text{if } i \in \bar{R}_N \end{cases}$$

The resultant proposed neutrosophic estimators under schemes I-III are provided as

$$T_{v_1} = \left[\alpha_1 \bar{y}_{r_N,rss} + \gamma_1 \bar{y}_{r_N,rss}\left\{\frac{\bar{X}_N}{\theta_N \bar{x}_{n_N,rss} + (1-\theta_N)\bar{X}_N}\right\}^g\right]\left\{1+\log\left(\frac{\bar{x}_{n_N,rss}}{\bar{X}_N}\right)\right\}^{\phi_1},$$
$$T_{v_2} = \left[\alpha_2 \bar{y}_{r_N,rss} + \gamma_2 \bar{y}_{r_N,rss}\left\{\frac{\bar{X}_N}{\theta_N \bar{x}_{r_N,rss} + (1-\theta_N)\bar{X}_N}\right\}^g\right]\left\{1+\log\left(\frac{\bar{x}_{r_N,rss}}{\bar{X}_N}\right)\right\}^{\phi_2},$$
$$T_{v_3} = \left[\alpha_3 \bar{y}_{r_N,rss} + \gamma_3 \bar{y}_{r_N,rss}\left\{\frac{\bar{x}_{n_N,rss}}{\theta_N \bar{x}_{r_N,rss} + (1-\theta_N)\bar{x}_{n_N,rss}}\right\}^g\right]\left\{1+\log\left(\frac{\bar{x}_{r_N,rss}}{\bar{x}_{n_N,rss}}\right)\right\}^{\phi_3},$$
$$T_{v_4} = \left\{\alpha_4 \bar{y}_{r_N,rss} + \gamma_4(\bar{x}_{n_N,rss} - \bar{X}_N)\right\} \times \left\{1+\log\left(\frac{\bar{x}_{n_N,rss}}{\bar{X}_N}\right)\right\}^{\phi_4},$$
$$T_{v_5} = \left\{\alpha_5 \bar{y}_{r_N,rss} + \gamma_5(\bar{x}_{r_N,rss} - \bar{X}_N)\right\} \times \left\{1+\log\left(\frac{\bar{x}_{r_N,rss}}{\bar{X}_N}\right)\right\}^{\phi_5},$$
$$T_{v_6} = \left\{\alpha_6 \bar{y}_{r_N,rss} + \gamma_6(\bar{x}_{r_N,rss} - \bar{x}_{n_N,rss})\right\} \times \left\{1+\log\left(\frac{\bar{x}_{r_N,rss}}{\bar{x}_{n_N,rss}}\right)\right\}^{\phi_6},$$

where the scalars being optimized are α_i, $i = 1, 2, ..., 6$, γ_i, and ϕ_i. Additionally, in order to develop various enhanced neutrosophic imputation techniques and the related resultant estimators, g and θ_N are constants that take real values.

Theorem 3.1. *For the resulting neutrosophic suggested estimators T_{v_i}, $i = 1, 2, ..., 6$, the minimum MSE is provided by*

$$min.MSE(T_{v_i}) = \bar{Y}_N^2\left(1 - \frac{A_i E_i^2 + B_i D_i^2 - 2C_i D_i E_i}{A_i B_i - C_i^2}\right) \tag{3.1}$$

where

$$A_1 = 1 + V_0^* + 2\phi_1(\phi_1 - 1)V_1 + 4\phi_1 V_{01},$$
$$B_1 = 1 + V_0^* + \{2\phi_1(\phi_1 - 1) + g(g+1)\theta_N^2 + g^2\theta_N^2 - 4\phi_1 g\theta_N\}V_1 + 4(\phi_1 - g\theta_N)V_{01},$$
$$C_1 = 1 + V_0^* + \left\{\frac{g(g+1)}{2}\theta_N^2 - 2\phi_1 g\theta_N + 2\phi_1(\phi_1 - 1)\right\}V_1 + 2(2\phi_1 - g\theta_N)V_{01},$$
$$D_1 = 1 + \left\{\frac{\phi_1(\phi_1 - 2)}{2}\right\}V_1 + \phi_1 V_{01},$$
$$E_1 = 1 + \left\{\frac{g(g+1)}{2}\theta_N^2 - \phi_1 g\theta_N + \frac{\phi_1(\phi_1 - 2)}{2}\right\}V_1 + (\phi_1 - g\theta_N)V_{01},$$
$$A_2 = 1 + V_0^* + 2\phi_2(\phi_2 - 1)V_1^* + 4\phi_2 V_{01}^*,$$
$$B_2 = 1 + V_0^* + \{2\phi_2(\phi_2 - 1) + g(g+1)\theta_N^2 + g^2\theta_N^2 - 4\phi_2 g\theta_N\}V_1^* + 4(\phi_2 - g\theta_N)V_{01}^*,$$
$$C_2 = 1 + V_0^* + \left\{\frac{g(g+1)}{2}\theta_N^2 - 2\phi_2 g\theta_N + 2\phi_2(\phi_2 - 1)\right\}V_1^* + 2(2\phi_2 - g\theta_N)V_{01}^*,$$
$$D_2 = 1 + \left\{\frac{\phi_2(\phi_2 - 2)}{2}\right\}V_1^* + \phi_2 V_{01}^*,$$

$$E_2 = 1 + \left\{ \frac{g(g+1)}{2}\theta_N^2 - \phi_2 g\theta_N + \frac{\phi_2(\phi_2-2)}{2} \right\} V_1^* + (\phi_2 - g\theta_N)V_{01}^*,$$
$$A_3 = 1 + V_0^* + 2\phi_3(\phi_3-1)(V_1^*-V_1) + 4\phi_3(V_{01}^*-V_{01}),$$
$$B_3 = 1 + V_0^* + \{2\phi_3(\phi_3-1) + g(g+1)\theta_N^2 + g^2\theta_N^2 - 4\phi_3 g\theta_N\}(V_1^*-V_1) + 4(\phi_3-g\theta_N)(V_{01}^*-V_{01}),$$
$$C_3 = 1 + V_0^* + \left\{ \frac{g(g+1)}{2}\theta_N^2 - 2\phi_3 g\theta_N + 2\phi_3(\phi_3-1) \right\} (V_1^*-V_1) + 2(2\phi_3-g\theta_N)(V_{01}^*-V_{01}),$$
$$D_3 = 1 + \left\{ \frac{\phi_3(\phi_3-2)}{2} \right\} (V_1^*-V_1) + \phi_3(V_{01}^*-V_{01}),$$
$$E_3 = 1 + \left\{ \frac{g(g+1)}{2}\theta_N^2 - \phi_3 g\theta_N + \frac{\phi_3(\phi_3-2)}{2} \right\} (V_1^*-V_1) + (\phi_3-g\theta_N)(V_{01}^*-V_{01}),$$
$$A_4 = 1 + V_0^* + 2\phi_4(\phi_4-1)V_1 + 4\phi_4 V_{01},$$
$$B_4 = V_1/R^2,$$
$$C_4 = (2\phi_4 V_1 + V_{01})/R,$$
$$D_4 = 1 + \left\{ \frac{\phi_4(\phi_4-2)}{2} \right\} V_1 + \phi_4 V_{01},$$
$$E_4 = \phi_4 V_1/R,$$
$$A_5 = 1 + V_0^* + 2\phi_5(\phi_5-1)V_1^* + 4\phi_5 V_{01}^*,$$
$$B_5 = V_1^*/R^2,$$
$$C_5 = (2\phi_5 V_1^* + V_{01}^*)/R,$$
$$D_5 = 1 + \left\{ \frac{\phi_5(\phi_5-2)}{2} \right\} V_1^* + \phi_5 V_{01}^*,$$
$$E_5 = \phi_5 V_{01}^*/R,$$
$$A_6 = 1 + V_0^* + 2\phi_6(\phi_6-1)(V_1^*-V_1) + 4\phi_6(V_{01}^*-V_{01}),$$
$$B_6 = (V_1^*-V_1)/R^2,$$
$$C_6 = \{2\phi_6(V_1^*-V_1) + (V_{01}^*-V_{01})\}/R,$$
$$D_6 = 1 + \left\{ \frac{\phi_6(\phi_6-2)}{2} \right\} (V_1^*-V_1) + \phi_6(V_{01}^*-V_{01}),$$
$$E_6 = \phi_6(V_1^*-V_1)/R.$$

Proof. Appendix A provides an outline of the proof. □

4. Theoretical comparisons

To evaluate efficiency, bias, and mean square error in survey sampling, theoretical comparisons of the suggested and modified imputation techniques and their accompanying estimators are essential. These comparisons ensure robustness and applicability in real-life surveys by illuminating the circumstances in which the suggested estimators perform better than the adapted ones. By offering a solid mathematical basis for suggesting better estimating methods, these comparisons raise the precision and dependability of survey findings. The MSE of the proposed and modified resultant neutrosophic estimators is compared under the following lemmas in order to conduct theoretical comparisons because of their significance.

Lemma 4.1. *The neutrosophic mean estimator T_m under schemes I-III is dominated by the suggested neutrosophic estimators T_{v_i}, $i = 1,2,...,6$, if*

$$min.MSE(T_{v_i}) < V(T_m) \implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^*.$$

Lemma 4.2. *(i). Under scheme I, the neutrosophic ratio estimator T_{r_1} is subordinated to the suggested neutrosophic estimators T_{v_i}, $i = 1,4$, if*

$$min.MSE(T_{v_i}) < MSE(T_{r_1})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0 - V_1 + 2V_{01}.$$

(ii). The proposed neutrosophic estimators T_{v_i}, $i = 2,5$ dominate the neutrosophic ratio estimator T_{r_2} under scheme II, if

$$min.MSE(T_{v_i}) < MSE(T_{r_2})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^* - V_1^* + 2V_{01}^*.$$

(iii). The proposed neutrosophic estimators T_{v_i}, $i = 3,6$ dominate the neutrosophic ratio estimator T_{r_3} under scheme III, if

$$min.MSE(T_{v_i}) < MSE(T_{r_3})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^* - (V_1^* - V_1) + 2(V_{01}^* - V_{01}).$$

Lemma 4.3. *(i). The suggested neutrosophic estimators T_{v_i}, $i = 1,4$ is the dominant factor in scheme I of the neutrosophic regression estimator T_{lr_1}, if*

$$min.MSE(T_{v_i}) < min.MSE(T_{lr_1})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^* + \frac{V_{01}^2}{V_1}.$$

(ii). The proposed neutrosophic estimators T_{v_i}, $i = 2,5$ dominate the neutrosophic regression estimator T_{lr_2} under scheme II, if

$$min.MSE(T_{v_i}) < min.MSE(T_{lr_2})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^* + \frac{{V_{01}^*}^2}{V_1^*}.$$

(iii). The proposed neutrosophic estimators T_{v_i}, $i = 3,6$ dominate the neutrosophic regression estimator T_{lr_3} under scheme III, if

$$min.MSE(T_{v_i}) < min.MSE(T_{lr_3})$$
$$\implies \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} > 1 - V_0^* + \frac{(V_{01}^* - V_{01})^2}{(V_1^* - V_1)}.$$

Lemma 4.4. *(i). The proposed neutrosophic estimators T_{v_i}, $i = 1,4$ dominate Searls type neutrosophic difference estimator T_{b_1} and Searls type neutrosophic ratio estimator T_{b_4} both envisaged on the lines of Bhushan et al. (2023) under scheme I, if*

$$\begin{aligned} min.MSE(T_{v_i}) &< min.MSE(T_{b_i}),\ i = 1,4 \\ \Longrightarrow \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} &> \frac{H_i^2}{G_i}. \end{aligned}$$

(ii). The proposed neutrosophic estimator T_{v_i}, $i = 2,5$ dominate Searls type neutrosophic difference estimator T_{b_2} and Searls type neutrosophic ratio estimator T_{b_5} both envisaged on the lines of Bhushan et al. (2023) under scheme II, if

$$\begin{aligned} min.MSE(T_{v_i}) &< min.MSE(T_{b_i}),\ i = 2,5 \\ \Longrightarrow \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} &> \frac{H_i^2}{G_i}. \end{aligned}$$

(iii). The proposed neutrosophic estimator T_{v_i}, $i = 3,6$ dominate Searls type neutrosophic difference estimator T_{b_3} and Searls type neutrosophic ratio estimator T_{b_6} both envisaged on the lines of Bhushan et al. (2023) under scheme III, if

$$\begin{aligned} min.MSE(T_{v_i}) &< min.MSE(T_{b_i}),\ i = 3,6 \\ \Longrightarrow \frac{A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i}{A_iB_i - C_i^2} &> \frac{H_i^2}{G_i}. \end{aligned}$$

5. Simulation study

Simulation is useful in survey research because it allows researchers to examine and enhance the developed estimation procedures. It is useful for determining the impact of missing data, testing imputation methods, and analysing potential biases. In this section, following Singh and Horn (1998) and Kumar and Kumar (2025c), a simulation study is performed using a neutrosophic normal population and a neutrosophic Weibull population. The data for these populations are generated using the following models:

$$\left.\begin{aligned} y_N &= 12.9 + \sqrt{(1-\rho_{xy_N}^2)}\,y_N^* + \rho_{xy_N}\left(\frac{S_{y_N}}{S_{x_N}}\right)x_N^* \\ \text{and}\ \ x_N &= 12.5 + x_N^*. \end{aligned}\right\} \tag{5.1}$$

Neutrosophic variables x_N^* and y_N^* in the models given in (5.1) are based on different neutrosophic distributions. The simulation study is explained in the following points.

(i). The models in (5.1) are used to generate the normal and Weibull populations. The details of these populations are given as follows:
(a). Generate a normal population from neutrosophic normal distribution (NND) with $x_N^* \sim NND([19,\ 22],\ [28,\ 30])$, $y_N^* \sim NND([16,\ 18],\ [30,\ 32])$, and several values of neutrosophic correlation coefficient $\rho_{xy_N} \in ([0.1,\ 0.2],\ [0.3,\ 0.4],\ [0.5,\ 0.6],\ [0.7,\ 0.8])$.
(b). Generate a Weibull population from neutrosophic Weibull distribution (NWD) of size $N = 1200$ with $x_N^* \sim NWD([0.16,\ 0.18],\ [0.22,\ 0.24])$, $y_N^* \sim NWD([0.14,\ 0.16],\ [0.24,\ 0.26])$, and several values of neutrosophic correlation coefficient $\rho_{xy_N} \in ([0.1,\ 0.2],\ [0.3,\ 0.4],\ [0.5,\ 0.6]$ $,\ [0.7,\ 0.8])$.

(ii). Select a neutrosophic ranked set sample of size $n_N = [12,\ 12]$ with set size $m_N = [3,3]$ and number of cycles $k = 4$ from the neutrosophic normal and neutrosophic Weibull populations generated in point (i).

(iv). Based on 10,000 iterations, compute the $MSE_N \in [MSE_L, MSE_U]$ and $PRE_N \in [PRE_L, PRE_U]$ of the resulting neutrosophic estimators for the responding probability $P_N = [0.67,\ 0.67]$ and the samples n_N taken in the previous step utilizing the respective expression provided in (5.2).

$$\left.\begin{aligned} MSE(T^*) &= \frac{1}{10,000}\sum_{j=1}^{10,000}(T_j^* - \bar{Y}_N)^2 \\ \text{and}\ \ PRE &= \frac{V(\bar{y}_{r_N,rss})}{MSE(T_j^*)}\times 100, \end{aligned}\right\} \tag{5.2}$$

where $T_j^* = T_m,\ T_{r_i},\ i = 1,2,3,\ T_{lr_i},\ T_{b_i},\ i = 1,2,...,6,\ T_{v_i},\ i = 1,2,...,6.$

The simulation codes may be provided to the reader upon reasonable request to the corresponding author. Tables 1 and 2 show the simulated $MSE_N \in [MSE_L, MSE_U]$, while Tables 3 and 4 show the simulated $PRE_N \in [PRE_L, PRE_U]$ for neutrosophic normal and neutrosophic Weibull populations, respectively.

5.1 Discussion of simulation findings

In order to have better understanding about the performance of the resulting neutrosophic estimators, we discussed the simulation findings reported from Tables 1-4 in the following points:

(i). The neutrosophic normal and neutrosophic Weibull populations' MSE_N is shown in Tables 1 and 2, respectively. The results under schemes I-III demonstrate that the recommended resulting neutrosophic estimators $T_{v_i},\ i = 1,2,...,6$ achieve the lowest MSE_N for each chosen value of ρ_{xy_N} when compared to the adapted resulting neutrosophic estimators T_m, neutrosophic ratio and regression estimators $T_{r_i},\ i = 1,2,3$, and T_{lr_i}, and Bhushan *et al.* (2023) type neutrosophic estimators $T_{b_i},\ i = 1,2,...,6$. This demonstrate the improved performance and accuracy of the suggested neutrosophic imputation methods over the adapted ones, while processing imprecise data.

(ii). Tables 3 and 4 exhibit PRE_N for the neutrosophic normal and neutrosophic Weibull populations, respectively. Under schemes I-III, the findings show that the suggested resulting neutrosophic estimators $T_{v_i},\ i = 1,2,...,6$ achieve the highest PRE_N for each selected value of ρ_{xy_N} compared to the adapted resulting neutrosophic estimators such as neutrosophic mean estimator T_m, neutrosophic ratio and regression estimators $T_{r_i},\ i = 1,2,3$ and T_{lr_i}, and Bhushan *et al.* (2023) type neutrosophic estimators $T_{b_i},\ i = 1,2,...,6$. These findings demonstrate the improved performance and accuracy of the suggested neutrosophic imputation methods over the adapted ones while dealing with missingness in imprecise data.

(iii). Tables 1 and 2 demonstrate that, for any value of ρ_{xy_N}, the MSE_N of the resulting neutrosophic mean estimator T_m is essentially stable. However, under schemes I-III, the MSE_N of the adapted resulting neutrosophic ratio and regression estimators $T_{r_i},\ i = 1,2,3$ and T_{lr_i}, Bhushan *et al.* (2023) type neutrosophic estimators $T_{b_i},\ i = 1,2,...,6$, and the proposed neutrosophic estimators $T_{v_i},\ i = 1,2,...,6$ steadily reduces as ρ_{xy_N} varies from minimum to maximum value. Furthermore, an opposite behaviour can be seen in PRE_N of all resulting neutrosophic estimators reported in Tables 3-4.

(v). In both populations, the suggested resulting neutrosophic estimators perform much superior in scheme II than in schemes I and III. This improved superiority of the proposed estimators in scheme II demonstrates that they are particularly well-suited for scenarios in which certain underlying conditions closely match the neutrosophic characteristics modelled in this class.

Table 1. $MSE_N \in [MSE_L,\ MSE_U]$ of the neutrosophic estimators using neutrosophic normal population

	ρ_{xy_N}			
Estimators	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[105.47, 119.52]	[104.66, 118.69]	[104.04, 118.17]	[103.80, 118.25]
Scheme I				
T_{r_1}	[147.55, 155.83]	[127.37, 130.19]	[101.43, 98.21]	[70.64, 61.09]
T_{lr_1}	[102.91, 113.20]	[94.20, 99.06]	[78.11, 76.64]	[54.81, 46.04]
T_{b_1}	[89.43, 100.65]	[84.94, 90.96]	[72.43, 72.14]	[51.97, 44.13]
T_{b_4}	[89.06, 99.94]	[84.17, 90.13]	[71.77, 71.58]	[51.62, 43.79]
T_{v_1}	[87.72, 97.89]	[81.81, 87.14]	[68.74, 68.15]	[48.51, 40.77]
T_{v_4}	[88.35, 98.48]	[82.28, 87.54]	[69.03, 68.37]	[48.64, 40.68]
Scheme II				
T_{r_2}	[168.28, 173.71]	[138.56, 135.86]	[100.14, 88.38]	[54.30, 32.93]
T_{lr_2}	[101.64, 110.08]	[89.04, 89.39]	[65.33, 56.18]	[30.68, 10.47]
T_{b_2}	[88.44, 98.11]	[80.62, 82.56]	[61.06, 53.32]	[29.19, 9.38]
T_{b_5}	[87.90, 97.06]	[79.49, 81.35]	[60.09, 52.39]	[28.32, 9.14]
T_{v_2}	[85.81, 93.91]	[75.89, 76.90]	[55.74 47.82]	[24.61, 5.26]
T_{v_5}	[86.82, 94.88]	[76.70, 77.60]	[56.27, 48.21]	[24.88, 4.40]
Scheme III				
T_{r_3}	[126.20, 137.41]	[115.85, 124.36]	[102.76, 108.34]	[87.47, 90.10]
T_{lr_3}	[104.21, 116.41]	[99.50, 109.02]	[91.27, 97.71]	[79.67, 82.69]
T_{b_3}	[90.44, 103.23]	[89.31, 99.44]	[83.80, 90.85]	[74.35, 77.90]
T_{b_6}	[90.26, 102.88]	[88.92, 99.01]	[83.47, 90.58]	[74.24, 77.93]
T_{v_3}	[89.62, 101.90]	[87.78, 97.53]	[81.92, 88.73]	[72.40, 75.85]
T_{v_6}	[89.91, 102.16]	[87.99, 97.70]	[82.03, 88.80]	[72.43, 75.85]

Table 2. $MSE_N \in [MSE_L, MSE_U]$ of the neutrosophic estimators using neutrosophic Weibull population

	ρ_{xy_N}			
Estimators	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[6033199.00, 284661.80]	[6010295.00, 283233.30]	[5974129.00, 281093.20]	[5926407.00, 278320.40]
Scheme I				
T_{r_1}	[7403271.00, 319327.50]	[6443014.00, 278536.90]	[5407441.00, 232335.60]	[4256552.00, 178982.10]
T_{lr_1}	[5939181.00, 272798.30]	[5511217.00, 242961.30]	[4690293.00, 194882.30]	[3501263.00, 130013.40]
T_{b_1}	[2685482.00, 129357.50]	[2624679.00, 123540.80]	[2429451.00, 109618.40]	[2045830.00, 83659.56]
T_{b_4}	[2644725.00, 126934.70]	[2552032.00, 122308.30]	[2408262.00, 112972.60]	[2132497.00, 87042.41]
T_{v_1}	[1837017.00, 94836.39]	[1643987.00, 84665.85]	[1388988.00, 69833.20]	[1029821.00, 44474.77]
T_{v_4}	[2551850.00, 118152.30]	[2210653.00, 100282.40]	[1720274.00, 74471.26]	[1116788.00, 38560.78]
Scheme II				
T_{r_2}	[8078083.00, 336401.60]	[6656144.00, 276223.70]	[5128326.00, 208320.70]	[3434085.00, 130054.30]
T_{lr_2}	[5892874.00, 266955.10]	[5265402.00, 223125.80]	[4057955.00, 152420.20]	[2306789.00, 56966.70]
T_{b_2}	[2675966.00, 128004.80]	[2565935.00, 117976.80]	[2238865.00, 93711.08]	[1523223.00, 38531.00]
T_{b_5}	[2615058.00, 124387.80]	[2457534.00, 116131.20]	[2204259.00, 97403.14]	[1810829.00, 61303.98]
T_{v_2}	[1107090.00, 68417.89]	[913583.10, 54341.81]	[574608.00, 32690.35]	[197619.10, 13082.49]
T_{v_5}	[2476591.00, 111331.20]	[1952091.00, 83822.87]	[1211179.00, 41740.28]	[245636.30, 19000.67]
Scheme III				
T_{r_3}	[6708011.00, 301735.90]	[6223425.00, 280920.20]	[5695014.00, 257078.30]	[5103941.00, 229392.60]
T_{lr_3}	[5986892.00, 278818.60]	[5764481.00, 263397.80]	[5341792.00, 238631.10]	[4731933.00, 205273.70]
T_{b_3}	[2695158.00, 130707.40]	[2681748.00, 128750.70]	[2598449.00, 122771.30]	[2432365.00, 111867.30]
T_{b_6}	[2675111.00, 129514.60]	[2645976.00, 128136.00]	[2587848.00, 124517.10]	[2483622.00, 117470.50]
T_{v_3}	[2338190.00, 115541.50]	[2244073.00, 110972.40]	[2112847.00, 103737.40]	[1930922.00, 92924.83]
T_{v_6}	[2629317.00, 125172.00]	[2476503.00, 117108.00]	[2240467.00, 104543.50]	[1911764.00, 86611.53]

Table 3. $PRE_N \in [PRE_L,\ PRE_U]$ of the neutrosophic estimators using neutrosophic normal population

	ρ_{xy_N}			
Estimators	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[100, 100]	[100, 100]	[100, 100]	[100, 100]
Scheme I				
T_{r_1}	[71.48, 76.70]	[82.16, 91.16]	[102.57, 120.32]	[146.95, 193.57]
T_{lr_1}	[102.49, 105.58]	[111.10, 119.81]	[133.20, 154.18]	[189.36, 256.82]
T_{b_1}	[117.93, 118.74]	[123.21, 130.47]	[143.64, 163.80]	[199.71, 267.93]
T_{b_4}	[118.42, 119.59]	[124.34, 131.68]	[144.96, 165.08]	[201.09, 270.04]
T_{v_1}	[120.23, 122.09]	[127.93, 136.20]	[151.35, 173.40]	[213.97, 290.04]
T_{v_4}	[119.37, 121.36]	[127.19, 135.58]	[150.71, 172.84]	[213.41, 290.63]
Scheme II				
T_{r_2}	[62.67, 68.80]	[75.53, 87.36]	[103.89, 133.70]	[191.16, 359.08]
T_{lr_2}	[103.76, 108.57]	[117.53, 132.78]	[159.24, 210.32]	[338.26, 1128.42]
T_{b_2}	[119.25, 121.82]	[129.81, 143.75]	[170.38, 221.63]	[355.60, 1260.65]
T_{b_5}	[119.99, 123.14]	[131.65, 145.89]	[173.14, 225.53]	[366.50, 1293.76]
T_{v_2}	[122.91, 127.27]	[137.89, 154.32]	[186.64, 247.09]	[421.80, 2246.33]
T_{v_5}	[121.48, 125.97]	[136.45, 152.95]	[184.88, 245.11]	[417.21, 2687.56]
Scheme III				
T_{r_3}	[83.57, 86.98]	[90.34 95.44]	[101.25, 109.07]	[118.67, 131.25]
T_{lr_3}	[101.21, 102.67]	[105.17, 108.86]	[113.99, 120.93]	[130.28, 143.01]
T_{b_3}	[116.61, 115.77]	[117.18, 119.36]	[124.14, 130.06]	[139.60, 151.79]
T_{b_6}	[116.85, 116.17]	[117.69, 119.87]	[124.64, 130.45]	[139.82, 151.73]
T_{v_3}	[117.68, 117.29]	[119.22, 121.68]	[127.00, 133.18]	[143.36, 155.89]
T_{v_6}	[117.30, 116.99]	[118.94, 121.48]	[126.82, 133.06]	[143.30, 155.90]

Table 4. $PRE_N \in [PRE_L,\ PRE_U]$ of the neutrosophic estimators using neutrosophic Weibull population

	ρ_{xy_N}			
Estimators	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[100, 100]	[100, 100]	[100, 100]	[100, 100]
Scheme I				
T_{r_1}	[81.49, 89.14]	[93.28, 101.68]	[110.47, 120.98]	[139.23, 155.50]
T_{lr_1}	[101.58, 104.34]	[109.05, 116.57]	[127.37, 144.23]	[169.26, 214.07]
T_{b_1}	[224.65, 220.05]	[228.99, 229.26]	[245.90, 256.42]	[289.68, 332.68]
T_{b_2}	[228.12, 224.25]	[235.51, 231.57]	[248.06, 248.81]	[277.90, 319.75]
T_{v_1}	[328.42, 300.16]	[365.59, 334.53]	[430.10, 402.52]	[575.47, 625.79]
T_{v_4}	[236.42, 240.92]	[271.87, 282.43]	[347.27, 377.45]	[530.66, 721.77]
Scheme II				
T_{r_2}	[74.68, 84.61]	[90.29, 102.53]	[116.49, 134.93]	[172.57, 214.00]
T_{lr_2}	[102.38, 106.63]	[114.14, 126.93]	[147.22, 184.41]	[256.91, 488.56]
T_{b_2}	[225.45, 222.38]	[234.23, 240.07]	[266.83, 299.95]	[389.07, 722.32]
T_{b_5}	[230.71, 228.85]	[244.56, 243.89]	[271.02, 288.58]	[327.27, 454.00]
T_{v_2}	[544.96, 416.06]	[657.88, 521.20]	[1039.68, 859.86]	[1622.76, 2127.42]
T_{v_5}	[243.60, 255.68]	[307.89, 337.89]	[493.24, 673.43]	[512.67, 671.72]
Scheme III				
T_{r_3}	[89.94, 94.34]	[96.57, 100.82]	[104.90, 109.34]	[116.11, 121.32]
T_{lr_3}	[100.77, 102.09]	[104.26, 107.53]	[111.83, 117.79]	[125.24, 135.58]
T_{b_3}	[223.85, 217.78]	[224.11, 219.98]	[229.91, 228.95]	[243.64, 248.79]
T_{b_6}	[225.53, 219.79]	[227.14, 221.04]	[230.85, 225.74]	[238.61, 236.92]
T_{v_3}	[258.02, 246.37]	[267.82, 255.22]	[282.75, 270.96]	[306.92, 299.51]
T_{v_6}	[229.45, 227.41]	[242.69, 241.85]	[266.64, 268.87]	[309.99, 321.34]

6. Real data application

Real data insights are important in survey sampling because they give an accurate depiction of population features, establishing a connection between conceptual models and practical applications. It is an open source dataset that includes the daily stock price of "Moderna" https://finance.yahoo.com/quote/MRNA/history/. From 1 September 2020 to 1 September 2021, the neutrosophic survey variable $y_N \in [y_L,\ y_U]$ represents the stock's daily fluctuating price, and the neutrosophic auxiliary variable $x_N \in [x_L,\ x_U]$ represents the stock's daily fluctuating price from 1 September 2019 to 1 September 2020. Table 5 reports the descriptive statistics needed to compute the $MSE_N \in [MSE_L,\ MSE_U]$ and $PRE_N \in [PRE_L,\ PRE_U]$. Utilizing these descriptive, we have calculated MSE_N and PRE_N for the resulting neutrosophic estimators and reported the findings in Table 6. The PRE_N is calculated utilizing the formula given below.

$$PRE_N = \frac{V(\bar{y}_{r_N,rss})}{MSE(T^*)} \times 100, \tag{6.1}$$

where T^* represents the resulting adapted and suggested neutrosophic estimators.

The findings of real data reported in Table 6 exhibit that the MSE_N of the proposed neutrosophic estimators $T_{v_i},\ i = 1, 2, ..., 6$ is minimum compared to the MSE_N of the adapted neutrosophic estimators such as neutrosophic sample mean $\bar{y}_N$, neutrosophic ratio and regression estimators $T_{r_i},\ i = 1, 2, 3$ and T_{lr_i}, and Bhushan *et al.* (2023) type neutrosophic estimators $T_{b_i},\ i = 1, 2, ..., 6$. Furthermore, the real data findings of Table 6 exhibit that the PRE_N of the suggested neutrosophic estimators

$T_{v_i},\ i = 1, 2, ..., 6$ is highest compared to the PRE_N of the adapted neutrosophic estimators. Additionally, in scheme II, the performance of the proposed and modified neutrosophic estimators is significantly better than in schemes I and III.

Table 5. Descriptive statistics of real dataset

Desriptive statistics	Dataset
N_N	[252, 252]
n_N	[12, 12]
m_N	[3, 3]
k	4
P_N	[0.67, 0.67]
$\bar{Y}_N$	[162.973, 174.228]
$\bar{X}_N$	[35.94, 38.762]
C_{y_N}	[0.547, 0.559]
C_{x_N}	[0.603, 0.607]
ρ_{xy_N}	[0.829, 0.804)

Table 6. MSE_N and PRE_N of neutrosophic estimators for real dataset

Estimators	$[MSE_L, MSE_U]$	$[PRE_L, PRE_U]$
T_m	[2009.32, 2337.18]	[100.00, 100.00]
Scheme I		
T_{r_1}	[2207.91, 2553.52]	[91.00, 91.52]
T_{lr_1}	[1908.74, 2206.90]	[105.26, 105.90]
T_{b_1}	[1817.40, 2102.26]	[110.55, 111.17]
T_{b_4}	[1809.35, 2092.47]	[111.05, 111.69]
T_{v_1}	[1793.87, 2073.28]	[112.01, 112.72]
T_{v_4}	[1795.72, 2075.48]	[111.89, 112.60]
Scheme II		
T_{r_2}	[2305.71, 2660.08]	[87.14, 87.86]
T_{lr_2}	[1859.20, 2142.74]	[108.07, 109.07]
T_{b_2}	[1772.44, 2043.96]	[113.36, 114.34]
T_{b_5}	[1760.57, 2029.57]	[114.12, 115.15]
T_{v_2}	[1737.10, 2000.47]	[115.67, 116.83]
T_{v_5}	[1740.02, 2003.95]	[115.47, 116.62]
Scheme III		
T_{r_3}	[2107.13, 2443.74]	[95.35, 95.63]
T_{lr_3}	[1959.78, 2273.01]	[102.52, 102.82]
T_{b_3}	[1863.62, 2162.17]	[107.81, 108.09]
T_{b_6}	[1859.59, 2157.26]	[108.05, 108.33]
T_{v_3}	[1852.10, 2147.97]	[108.48, 108.80]
T_{v_6}	[1852.95, 2148.99]	[108.43, 108.75]

7. Conclusions

In survey sampling, the conventional imputation methods frequently struggle to deal with the missing values in uncertain and imprecise data, producing biassed and sometimes misleading results. On the other hand, the neutrosophic imputation methods based on neutrosophic logic, offer a more complete approach by taking into account three critical dimensions namely, truth, indeterminacy, and falsity. Some effective neutrosophic imputation techniques were presented in this study, together with the resulting efficient neutrosophic population mean estimators and their first-order estimated attributes, including bias and MSE. Under the obtained theoretical assumptions, the proposed neutrosophic imputation methods consistently yield more accurate and trustworthy estimates than the adapted neutrosophic imputation methods. A comprehensive simulation investigation using neutrosophic normal and neutrosophic Weibull datasets validated the theoretical findings. According to the simulation results, the suggested neutrosophic imputations performed better than the modified ones. Additionally, the results of a practical application utilizing imprecise stock price data show how applicable and useful neutrosophic imputation techniques are, highlighting their significance in enhancing the caliber and dependability of statistical inference. Thus, the proposed neutrosophic imputation methods may be recommended to estimate the population mean in the case of missing values in imprecise data.

In forthcoming studies, this article may be extended for the population proportion estimation under neutrosophic environment using RSS and extreme RSS (e.g., Zamanzadeh and Wang (2018) and Zamanzadeh and Mahdizadeh (2020)).

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Data availability

The real data used in this study can be obtained from open source database https://finance.yahoo.com/quote/MRNA/history/.

Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: KUMAR, A. **Data curation**: KUMAR, V. **Formal analysis**: KUMAR, V. **Investigation**: KUMAR, V.; KUMAR, A. **Methodology**: KUMAR, A. **Software**: KUMAR, V. **Resources**: KUMAR, V.; KUMAR, A. **Supervision**: KUMAR, A. **Validation**:KUMAR, V.; KUMAR, A. **Visualization**: KUMAR, V.; KUMAR, A. **Writing – original draft**: KUMAR, V.; KUMAR, A. **Writing – review and editing**: KUMAR, V.; KUMAR, A.

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Appendix A

Proof. Consider the first proposed resulting neutrosophic estimator under scheme I as

$$T_{\nu_1} = \left[\alpha_1 \bar{y}_{r_N,rss} + \gamma_1 \bar{y}_{r_N,rss} \left\{ \frac{\bar{X}_N}{\theta_N \bar{x}_{n_N,rss} + (1-\theta_N)\bar{X}_N} \right\}^g \right] \left\{ 1 + \log\left(\frac{\bar{x}_{n_N,rss}}{\bar{X}_N} \right) \right\}^{\phi_1}$$

Utilizing the notations given in (1.1), we can express the above estimator as

$$T_{\nu_1} - \bar{Y}_N = \bar{Y}_N \left[\begin{array}{l} \alpha_1 \left\{ 1 + \epsilon_{0N} + \phi_1 \epsilon_{2N} + \phi_1 \epsilon_{0N}\epsilon_{2N} + \dfrac{\phi_1(\phi_1-2)}{2}\epsilon_{2N}^2 \right\} \\ +\gamma_1 \left\{ \begin{array}{l} 1 + \epsilon_{0N} + \phi_1 \epsilon_{2N} - g\theta_N \epsilon_{2N} - g\theta_N \epsilon_{0N}\epsilon_{2N} \\ -\phi_1 g\theta_N \epsilon_{2N}^2 + \phi_1 \epsilon_{0N}\epsilon_{2N} + \dfrac{g(g+1)}{2}\theta_N^2 \epsilon_{2N}^2 \\ +\dfrac{\phi_1(\phi_1-2)}{2}\epsilon_{2N}^2 \end{array} \right\} - 1 \end{array} \right] \tag{A.1}$$

Taking expectation both the sides of (A.1), we get the bias of the proposed neutrosophic estimator as

$$Bias(T_{\nu_1}) = \bar{Y}_N \left[\begin{array}{l} \alpha_1 \left\{ 1 + \phi_1 V_{01} + \dfrac{\phi_1(\phi_1-2)}{2} V_1 \right\} \\ +\gamma_1 \left\{ \begin{array}{l} \left\{ \dfrac{g(g+1)}{2}\theta_N^2 - \phi_1 g\theta_N + \dfrac{\phi_1(\phi_1-2)}{2} \right\} V_1 \\ +(\phi_1 - g\theta_N) V_{01} \end{array} \right\} - 1 \end{array} \right] \tag{A.2}$$

The above expression can further be expressed as

$$Bias(T_{\nu_1}) = \bar{Y}_N(\alpha_1 D_1 + \gamma_1 E_1 - 1) \tag{A.3}$$

where

$$\begin{aligned} D_1 &= 1 + \frac{\phi_1(\phi_1-2)}{2} V_1 + \phi_1 V_{01}, \\ E_1 &= 1 + \left\{ \frac{g(g+1)}{2}\theta_N^2 - \phi_1 g\theta_N + \frac{\phi_1(\phi_1-2)}{2} \right\} V_1 + (\phi_1 - g\theta_N) V_{01}. \end{aligned}$$

Again, squaring and taking expectation both sides to (A.1), we can get the proposed estimator's MSE up to the first order approximation as

$$MSE(T_{\nu_1}) = \bar{Y}_N^2 \left[\begin{array}{l} 1 + \alpha_1^2 \left\{ 1 + V_0^* + 2\phi_1(\phi_1-1)V_1 + 4\phi_1 V_{01} \right\} \\ +\gamma_1^2 \left\{ \begin{array}{l} 1 + V_0^* + \left(\begin{array}{l} 2\phi_1(\phi_1-1) + g(g+1)\theta_N^2 \\ +g^2\theta_N^2 - 4\phi_1 g\theta_N \end{array} \right) V_1 \\ +4(\phi_1 - g\theta_N)V_{01} \end{array} \right\} \\ +2\alpha_1\gamma_1 \left\{ \begin{array}{l} 1 + V_0^* + \left\{ \begin{array}{l} \dfrac{g(g+1)}{2}\theta_N^2 - 2\phi_1 g\theta_N \\ +2\phi_1(\phi_1-1) \end{array} \right\} V_1 \\ +4(\phi_1 - g\theta_N)V_{01} \end{array} \right\} \\ -2\alpha_1 \left\{ 1 + \dfrac{\phi_1(\phi_1-2)}{2} V_1 + \phi_1 V_{01} \right\} \\ -2\gamma_1 \left\{ \begin{array}{l} 1 + \left(\dfrac{g(g+1)}{2}\theta_N^2 - \phi_1 g\theta_N + \dfrac{\phi_1(\phi_1-2)}{2} \right) V_1 \\ +(\phi_1 - g\theta_N)V_{01} \end{array} \right\} \end{array} \right] \tag{A.4}$$

The above expression can further be expressed as

$$MSE(T_{v_1}) = \bar{Y}_N^2(1+\alpha_1^2 A_1 + \gamma_1^2 B_1 + 2\alpha_1\gamma_{1N}C_1 - 2\alpha_1 D_1 - 2\gamma_1 E_1) \tag{A.5}$$

where,

$$\begin{aligned}
A_1 &= 1 + V_0^* + 2\phi_1(\phi_1 - 1)V_1 + 4\phi_1 V_{01},\\
B_1 &= \left[\begin{array}{l} 1 + V_0^* + \left(2\phi_1(\phi_1-1) + g^2\theta_N^2 + g(g+1)\theta_N^2 - 4g\theta_N\phi_1\right) V_1 \\ +4(\phi_1 - g\theta_N)V_{01} \end{array}\right],\\
C_1 &= \left[\begin{array}{l} 1 + V_0^* + \left\{\dfrac{g(g+1)}{2}\theta_N^2 + 2\phi_1(\phi_1-1) - 2\phi_1 g\theta_N\right\} V_1 \\ +4(\phi_1 - g\theta_N)V_{01} \end{array}\right],\\
D_1 &= 1 + \frac{\phi_1(\phi_1-2)}{2}V_1 + \phi_1 V_{01},\\
E_1 &= \left[\begin{array}{l} 1 + \left(\dfrac{g(g+1)}{2}\theta_N^2 - \phi_1 g\theta_N + \dfrac{\phi_1(\phi_1-2)}{2}\right) V_1 \\ +(\phi_1 - g\theta_N)V_{01} \end{array}\right].
\end{aligned}$$

The optimum values of scalars α_1, γ_1, and ϕ_1 are given by

$$\begin{aligned}
\alpha_{1(opt)} &= \frac{B_1 D_1 - C_1 E_1}{A_1 B_1 - C_1^2},\\
\gamma_{1(opt)} &= \frac{A_1 E_1 - C_1 D_1}{A_1 B_1 - C_1^2},\\
\text{and}\quad \phi_{1(opt)} &= -\rho_{xy_N}\frac{C_{y_N}}{C_{x_N}}.
\end{aligned}$$

The minimum MSE can be obtained by putting optimum values of α_1 and γ_1 in (A.5) as

$$min.MSE(T_{v_1}) = \bar{Y}_N^2\left(1 - \frac{A_1 E_1^2 + B_1 D_1^2 - 2C_1 D_1 E_1}{A_1 B_1 - C_1^2}\right). \tag{A.6}$$

Similarly, we can find out the bias and MSE of the resulting neutrosophic estimators under schemes II and III.

□