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ARTICLE

Minimum cost trend free 16-run foldover designs[1]

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Abstract
This paper proposes new 16-run foldover designs aimed at minimizing the number of level changes. The method involves selecting n - p independent columns from the main effects and interaction effects of a 2^4 full factorial experiment while avoiding run duplication to construct a 2^{n-p} fractional factorial design. The remaining p columns are then generated using these selected independent columns to maintain cost efficiency. The number of level changes and trend-free factors are computed for all possible fold-over plans for both standard and new designs. The performance of the new designs is compared to standard designs proposed by Li and Lin (2003), Cheng and Steinberg (1991) and Coster (1993) using the criteria of minimum level change, maximum trend-free factors and uniformity exhibiting improved performance and cost-effectiveness.

Keywords: Fractional factorial design, fold-over designs, number of level changes, trend free factor, uniformity measures.

1. Introduction

Factorial experiments are widely used across disciplines such as engineering, chemistry, agriculture, and industry to examine the effects of various factor combinations on a given response or outcome. However, conducting a full factorial experiment, which involves testing all possible combinations of factor(s) can take a lot of time and resources. To address this, fractional factorial designs (FFDs) are often used as a more efficient alternative, significantly reducing the required number of experimental runs. The main difficulty is selecting an appropriate fraction that aligns with available resources while maintaining the reliability and statistical validity of the results. The successful use of FFDs depends on finding this balance between cost-effectiveness and reliability of results.

In the design and execution of experiments, especially those involving fractional factorial plans, the order in which treatment combinations are implemented plays a critical role. In many cases, researchers may suspect potential temporal trend factors, such as aging effects, environmental fluctuations, or other time-dependent influences that may have a systematic effect on the experimental outcomes. When such trends are suspected, assigning treatments in a purely random sequence may compromise the reliability of the results. In these cases, adopting a more structured or balanced sequencing technique can help mitigate the effects and consequences of temporal trend. This, in turn, leads to more precise estimates of main effects and two-factor interactions, thereby improving the overall validity of the experimental results.

In addition to temporal trends, another important factor in experimental design is the cost function associated with changing factor levels. In industrial and laboratory experiments, changing machine configurations, chemical concentrations, or environmental conditions/variables can incur substantial costs in both time and resources. As a result, minimizing the number of factor level changes becomes essential for increasing the efficiency of the experiment. Optimal designs seek to balance these competing objectives: minimizing the total number of changes in the factor levels while simultaneously maximizing trend-freeness, ensuring both cost-effectiveness and robustness in the estimation of treatment effects.

1.1 Literature Review

The concept of systematic run orders in experimental design was first introduced by Cox (1951), and has since prompted substantial research for improving run sequences to balance trend resistance and cost-efficiency. They considered an example in which an experimenter compared the effect of several treatments when applied on wool. The wool is divided into lots, and each lot is given a specific treatment on a weekly basis. However, the experimenter suspects that the age of the wool may influence the processing results by introducing a smooth temporal trend. In such situations, random assignment of treatments may confound the results due to this underlying trend introduced by aging. To address this, trend-free designs, which assign treatments in a systematic order can be used to minimize the impact of time-related effects and produce more accurate comparisons.

Daniel and Wilcoxon (1966) identified specific 2^n factorial designs that are robust to both linear and quadratic trends, thereby ensuring the reliability of treatment comparisons in the presence of such effects. Draper and Stoneman (1968) suggested a methodology for establishing run sequence that minimizes the count of factor level changes, with the goal of reducing operational costs. Dickson (1974) further extended their work by generating minimum-cost run sequences for complete 2^4 and 2^5 factorial designs, which provided valuable insights into the trade-off between experimental efficiency and resource constraints.

Joiner and Campbell (1976) made further advances in the discipline by developing designs in which each factor changed levels with a defined probability, offering a probabilistic approach to managing level transitions. Cheng (1985) examined run orders that balanced trend resistance and cost minimization simultaneously. Subsequently, Coster and Cheng (1988) introduced a generalized fold-over scheme for constructing FFDs that are trend resistant.

Cheng and Jacroux (1988) considered run orders for both complete and fractional 2^n factorial designs, to maintain interpretability of the results even in the presence of temporal effects. This ensured that main effects and 2-fi’s remained orthogonal to polynomial trends. Cheng and Steinberg (1991) later proposed a reverse foldover algorithm, which improved earlier methods by purposely maximizing the number of level changes (NLC) while maintaining linear-trend-free designs. Major deficiency in the constructed designs was that although they were linear trend free designs but instead of minimizing the NLC to achieve cost efficiency they maximized NLC. Coster (1993) presented generator sequences for minimizing the cost function to produce systematic run order using GFS, which are trend free. Cheng *et al.* (1998), proposed algorithm for constructing two-level factorial designs with extreme (minimum and maximum) numbers of level changes. This approach yields design with resolution at least III or IV; however, their work did not address the robustness of factors effects in the presence of time trend, nor did it explore strategies for further reducing the number of level changes in the designs.

More recent studies have continued to make advances as far as the construction of trend-resistant and cost-efficient experimental designs is concerned. Adekeye and Kunert (2006) compared run orders in unreplicated 2^{n-p} designs with time trends. Singh, Thapliyal, and Budhraja (2013) constructed orthogonal arrays using linear codes with linear trend free run sequences. Singh, Thapliyal, and Budhraja (2014), extended this line of research by generating trend resistant orthogonal arrays using parity check matrix of linear codes and a simple straightforward methodology for constructing FFDs with s levels (where $s \geq 2$), ensuring that specified effects such as main effects and 2-fi's remained linear trend robust.

Singh, Thapliyal, and Budhraja (2016a, 2016b), also presented systematic methods for construction of linear trend resistant blocked factorial designs and FFDs using linear codes. Bhowmik *et al.* (2019, 2020) constructed response surface designs and cost-efficient factorial designs that minimize the NLC in run sequences, addressing both statistical robustness and practical feasibility. Thapliyal and Budhraja (2020) further contributed by developing trend-free run orders for 2-level factorial designs using symmetric BIBDs, thereby enriching the toolkit available for handling temporal trends in experimental designs.

The primary objective of identifying an optimal experimental design is to maximize the information gained while minimizing the number of experimental runs. One such technique is the uniform design, which was introduced by Fang (1980) and further developed by Wang and Fang (1981). Uniform designs are a type of space-filling designs that aim to distribute experimental points as evenly as possible across the experimental area. This uniformity is especially crucial in computer simulations, which are widely used in engineering and scientific applications where comprehensive domain coverage is essential.

To analyze the uniformity of a design, several discrepancy measures have been proposed, including star discrepancy, discrete discrepancy, L_p-discrepancy, symmetric L_p-discrepancy, centered L_2-discrepancy, and wrap-around L_2-discrepancy. Fang and Wang (1994) and Hickernell (1998a, 1998b), discussed various measures such as the centered L_2-discrepancy and wrap-around L_2-discrepancy for evaluating design uniformity.

Fang *et al.* (2002) established important theoretical connections between orthogonality, aberration, and uniformity, proving that these criteria work well in the context of 2- and 3- level factorial designs. Qin and Fang (2004) investigated the relationships between various measures of uniformity such as discrete discrepancy, generalized minimum aberration, minimum moment aberration, and the L_2-based discrepancies highlighting their interconnections and reinforcing the theoretical foundation for uniform experimental designs.

The paper is organized as follows: Section 2 discusses the preliminary results required to understand the concepts. Section 3 is methodology which, focuses on algorithm for construction of 16 run fold-over designs resulting in 32 run designs which have fewer numbers of level changes, have all main effects linear trend free (LTF) and few main effects quadratic trend free (QTF). Section 4 presents 2^{5-1}, 2^{6-2}, 2^{7-3} and 2^{8-4} foldover designs resulting in 32 run designs. These designs have been compared on the basis of NLC, LTF main effects, QTF main effects and uniformity with the standard fold-over designs proposed by Li and Lin (2003). Section 4 also compares newly constructed designs with the designs proposed by Cheng and Steinberg (1991) and Coster (1993). Section 5 presents the overall summary and conclusion.

2. Preliminaries

In a FFD, the numbers 1, 2, …, n associated with the factors are referred to as letters, and any product formed by a subset of these letters is known as a word. Let A_i represent the number of words of length *i* in the design's defining relation. The number of letters in a word is called its word length, and A(d) = (A_1, A_2, …, A_k) represents the word length pattern of the design.

2.1 Foldover designs

Box *et al.* 1978; Montgomery 2001 “defined a foldover design as a design obtained when a second fraction is added to the initial design by reversing the signs of one or more columns of the initial design”.

The NLC in a design is important when considering the cost of an experiment, as the cost of certain level changes in a design can be huge, particularly in industrial experiments, additionally, some level changes may be unnecessary for the design. Therefore, reducing the NLC in a design can lead to more cost-effective designs.

2.2 Number of level changes

Number of level change means number of times a factor changes its level from +1 to -1 i.e. to how many times the factor moves from the high level (+1) to the low level (-1) across different experimental conditions or runs. The design in which the number of level changes is less as compared to all other designs with the same parameters is known as the minimum level change design.

In some cases, experimental setups may be impacted by variations that can’t be identified or controlled, and these variations may be closely linked to the time or position at which they occur. When a trend is present in a fractional factorial experiment, it can influence the estimates of main effects and interactions. To avoid this, the order in which the experiments are run can be adjusted to make sure the trend doesn’t skew the results. A design where the treatment order is arranged in such a manner so as to prevent the trend from affecting the outcomes/responses, is called a trend-free design.

2.3 Trend free factors

Cheng and Jacroux (1988) “defined Time Count as let $y = (y_1, y_2, \dots, y_N)$ denotes the ordered vector of observations, $T_x = (1^x, 2^x, \dots, N^x)'$ for x = 0, 1, 2,…., v, v be the N × 1 vector of trend coefficients and let u_i be the contrast for main effect A_i ; i = 1, 2, ... n, in the run order. Then the quantity $u_i' T_x$ is known as the time count for the main effect A_i. A necessary and sufficient condition for a main effect contrast u to be v trend free is that

$$\boldsymbol{u_i' T_x = 0}, \qquad \forall \quad \boldsymbol{x = 1, 2, \dots, v} \qquad (1)$$

In general, an N × 1 vector u is called v trend free if (1) holds”.

2.4 Uniformity

The uniform experimental design (Fang, 1980; Wang and Fang, 1981) is an example of space filling design that allocates the design point uniformly spread in the experimental domain.

There are several different discrepancy measures among which centered L_2 – discrepancy (CD_2)

has been regarded more practicable and reasonable as they are invariant under reflection of the set of design points about any plane passing through the centre of the unit hypercube and parallel to its face. Hickernell (1998 b) gave analytical formula for the CD_2 as:

$$(CD_2)^2 = \left(\frac{13}{12}\right)^2 - \frac{2}{N}\sum_{k=1}^{N}\prod_{j=1}^{n}\left(1 + \frac{1}{2}\left|u_{kj} - 0.5\right| - \frac{1}{2}\left|u_{kj} - 0.5\right|^2\right) + \frac{1}{N^2}\sum_{k=1}^{N}\sum_{j=1}^{N}\prod_{i=1}^{n}\left(1 + \frac{1}{2}\left|u_{ki} - 0.5\right| + \frac{1}{2}\left|u_{ji} - 0.5\right| - \frac{1}{2}\left|u_{ki} - u_{ji}\right|\right) \quad (2)$$

3. Methodology

The run order of an experiment plays a critical role in minimizing the cost associated with factor level changes. When determining run order, experimenters must address two major challenges: (1) minimizing the number of level changes (NLC), which directly affects the effort and resources required to conduct the experiment, and (2) ensuring a trend-resistant run order to reduce the influence of time-related variables. Therefore, an optimal design aims to minimize level changes while maximizing the number of trend-free factors, thereby attaining both cost efficiency and statistical robustness.

In this study, several 16-run foldover designs are constructed with the objective of reducing level changes and increasing the number of factors that are LTF and QTF. These newly constructed designs are then compared with standard foldover designs (Li and Lin (2003)), and their performance is evaluated using centered L_2-discrepancy, a measure of design uniformity.

To construct the standard design, Li and Lin (2003) presented a 16-run 2^{n-p} FFD with specified generators, up to two-factor interactions for $5 \leq n \leq 15$. A corresponding set of 16 additional runs is generated by flipping the signs (i.e., reversing the levels) of one or more factors in the original design. These new runs are then appended to the original design to construct a foldover design, which serves as the benchmark or standard for comparison. All possible combinations of foldover columns are considered to exhaustively construct the foldover designs. For each design, the number of level changes and main effects that are linear and quadratic trend-free are calculated. Resolution of these designs is less than or equal to IV.

3.1. Algorithm for Constructing 16-Run Foldover Designs

To construct new improved 2^{n-p} FFDs, for $5 \leq n \leq 15$; $1 \leq p \leq 11$ a different technique is used.

Step 1: First a 2^4 full factorial design is generated including all main effects and 2-fi's to choose $n - p$ independent columns.

Step 2: From this design, Column 1, which exhibits the maximum NLC, is selected first, to avoid run duplication. Then, two more independent columns are selected from the main effects, each with relatively fewer NLC.

Step 3: An additional independent column is chosen from the two-factor interactions with the smallest NLC.

This selection results in a set of four independent columns that form the basis of new cost efficient fold over designs.

Step 4: After identifying the four independent columns, all possible combinations of these columns corresponding to two-, three-, and four-factor interactions are generated.

Step 5: From the systematically generated columns in the above mentioned step, '*p*' interaction columns exhibiting the minimum NLC are chosen through defining generator relationships to complete the design.

This ensures that the resulting design not only maintains statistical independence between the factors but also minimizes the overall NLC, which contributes to cost efficiency.

Step 6: A set of 16 additional runs is generated by flipping the signs of one or more factors in the design (foldover plan).

In general, the total number of foldover plans for a 2^{n-p} FFD would be $2^n – 1$.

Step 7: These new runs are then appended to the design produced in step 5 to construct a 16- run foldover design with 32 runs after ensuring that there is no run duplication due to fold over.

Step 8: In case there is run duplication the design may be dropped.

Step 9: From the generated 32 run designs, only those designs are selected which adhere to all linear trend free main effects condition.

Step10: For each design, the new constructions should be evaluated and compared against the corresponding standard designs in terms of:

1. Number of level changes, indicating experimental cost and operational simplicity.
2. Number of linear and quadratic trend free main effects, reflecting robustness against systematic trends.

It is also important to note that the 16 run designs generated have main effects (1, 3 and 4) which have not been excluded despite being non trend resistant. They do however become linear trend resistant after foldover. So, to get all linear trend resistant design the fold over plan must contain foldover of factors 1, 3 and 4. The number of designs generated considering foldover of remaining factors so that the resultant design is LTF for all factors is ${}^{p+1}C_0 + {}^{p+1}C_1 + {}^{p+1}C_2 + \ldots + {}^{p+1}C_{p+1} = (1 + 1)^{p+1} = 2^{p+1}$. Also, original design is of 16 runs and when the foldover plans corresponding to each of these runs such that the -1 signs can be flipped to +1 are considered they will result in 16 new runs with the one corresponding to the foldover plan yielding all 1’s run thereby leading to run duplicity as the original 16 runs have last run of all 1’s. Therefore, deleting all such fold over plans that result in run duplicity from 2^{p+1} remaining foldover plans give us the final desired designs. Thus, the number of designs remaining which have all effects LTF and do not have run duplicity are 2^{p+1}-2. In addition, foldover plans involving all factors result in a resolution IV design.

By systematically analyzing these metrics, we can determine the effectiveness of the newly constructed designs in improving efficiency over the standard designs.

Example 3.1: Consider a 2^{5-1} FFD. This design requires four independent columns and one generated column. First construct 2^4 full factorial experiment with all main effects and 2-fis.

Table 1. 2^4 full factorial experiment

S.N.	1*	2	3*	4*	12	13	14	23*	24	34
1	-1	-1	-1	-1	1	1	1	1	1	1
2	1	-1	-1	-1	-1	-1	-1	1	1	1
3	-1	1	-1	-1	-1	1	1	-1	-1	1
4	1	1	-1	-1	1	-1	-1	-1	-1	1
5	-1	-1	1	-1	1	-1	1	-1	1	-1
6	1	-1	1	-1	-1	1	-1	-1	1	-1
7	-1	1	1	-1	-1	-1	1	1	-1	-1

8	1	1	1	-1	1	1	-1	1	-1	-1
9	-1	-1	-1	1	1	1	-1	1	-1	-1
10	1	-1	-1	1	-1	-1	1	1	-1	-1
11	-1	1	-1	1	-1	1	-1	-1	1	-1
12	1	1	-1	1	1	-1	1	-1	1	-1
13	-1	-1	1	1	1	-1	-1	-1	-1	1
14	1	-1	1	1	-1	1	1	-1	-1	1
15	-1	1	1	1	-1	-1	-1	1	1	1
16	1	1	1	1	1	1	1	1	1	1
NLC	15	7	3	1	8	12	14	4	6	2

In this case, Columns A = 1, B = 3, C = 4, and D =23 are selected as the independent columns (as shown in Table 1). Among all possible interaction columns derived from these (i.e., all two-, three-, and four-factor interactions), Column BC = 34 is chosen as the generated column because it exhibits the fewest NLC (as shown in Table 2). This selection helps to maintain the necessary aliasing structure while minimizing the cost associated with factor level changes.

Table 2. Last row denotes the number of level changes.

A=1	B=3	C=4	D=23	AB	AC	AD	BC*	BD	CD	ABC	ABD	ACD	BCD	ABCD
-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1
1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1
-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1
-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1
1	1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1
-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1
-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1
1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1
-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1
1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1
-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1
1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	3	1	4	12	14	11	2	7	5	13	8	10	6	9

Suppose the columns of standard designs are denoted by 1, 2, 3, 4, … then the columns of new designs are A = 1, B = 3, C = 4, D = 23, E = BC = 34, ….

After identifying all the columns of new designs, consider the foldover plan in which the sign of factors A, B, C and E are reversed. Appending these 16 new runs generated by flipping the signs of factors A, B, C and E, to the generated design the resultant design is given in Table 3:

Table 3. New design with foldover plan ABCE

S.N.	A	B	C	D	E=BC
1	-1	-1	-1	1	1
2	1	-1	-1	1	1
3	-1	-1	-1	-1	1
4	1	-1	-1	-1	1
5	-1	1	-1	-1	-1
6	1	1	-1	-1	-1
7	-1	1	-1	1	-1
8	1	1	-1	1	-1
9	-1	-1	1	1	-1
10	1	-1	1	1	-1
11	-1	-1	1	-1	-1
12	1	-1	1	-1	-1
13	-1	1	1	-1	1
14	1	1	1	-1	1
15	-1	1	1	1	1
16	1	1	1	1	1
17	1	1	1	1	-1
18	-1	1	1	1	-1
19	1	1	1	-1	-1
20	-1	1	1	-1	-1
21	1	-1	1	-1	1
22	-1	-1	1	-1	1
23	1	-1	1	1	1
24	-1	-1	1	1	1
25	1	1	-1	1	1
26	-1	1	-1	1	1
27	1	1	-1	-1	1
28	-1	1	-1	-1	1
29	1	-1	-1	-1	-1
30	-1	-1	-1	-1	-1
31	1	-1	-1	1	-1
32	-1	-1	-1	1	-1
NLC	30	6	2	8	5
TC	0	0	0	0	0

Here NLC and TC are number of level changes and time count respectively.

In 2^{5-1} FFD with a fold-over plan 1235 = ABCE (mentioned in Table 4 marked with *), it is observed that the newly constructed design has fewer level changes (51) as compared to the standard design (75 from 5-1.2). Additionally, when analyzing about LTF components, the standard design exhibit four LTF main effects (1, 2, 3, and 5), whereas the newly constructed design ensures that all main effects are LTF.

Consider the foldover plans formulated by taking all possible combination of columns in 16 run FFD. For a 2^{5-1} design the foldover plans are:

A, B, C, D, E, AB, AC, AD, ………………………………………….., ABCDE.

The total number fold-over plans are $^5C_1 + {}^5C_2 + \ldots\ldots\ldots {}^5C_5 = 2^5 - 1$.

It is also important to note that the 16 run designs generated have main effects (1, 3 and 4) which have not been excluded despite being non trend resistant. They do however become linear trend resistant after foldover. So, to get all linear trend resistant design the foldover plan must contain foldover of factors 1, 3 and 4. The number of designs generated considering foldover of remaining factors so that the resultant design is LTF for all factors is, $^{1+1}C_0 + {}^2C_1 + {}^2C_2 = (1+1)^2 = 2^2$. Total number of foldover plans with all main effects LTF are ABC, ABCD, ABCE and ABCDE. Now Original 16 runs yield foldover plans ABC, BC, ABCD, BCD, ACDE, CDE, ACE, CE, ABE, BE, ABDE, BDE, AD, D which result in run duplicity when applied. Out of these ABC and ABCD are common with above-mentioned 4 foldover plans. Deleting the common ones as they result in run by duplication we are left with two designs ABCE and ABCDE which have all effects LTF and do not have run duplicity.

This approach can be applied to more complex designs. Specifically, new designs based on all possible fold-over plans for 2^{5-1}, 2^{6-2} and 2^{7-3} and 2^{8-4} designs (Table 4, 5, 6 and 7(b)) have been catalogued in this paper.

4. Results and discussions

This section contains newly constructed designs, their comparison with the standard designs given by Li and Lin (2003). Further the section compares the newly constructed designs with the designs constructed by Cheng and Steinberg (1991) and Coster (1993) to assess the newly constructed designs.

4.1 Comparison of newly constructed design with Standard design

Table 4 presents selected fold-over plans for the standard 2^{5-1} design given by Li and Lin (2003) and newly constructed fold-over plans in which all main effects are LTF.

Table 4. Standard and new designs for 2^{5-1}(n= 5 and p=1)

	Standard Design 2^{5-1}									New design 2^{5-1}			
Foldover plan	5-1.1 1,2,3,4,5=1234			5-1.2 1,2,3,4,5=12			5-1.3 1,2,3,4,5=12			A=1, B=3, C=4, D=23, E=BC			
	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	Foldover plan	NLC	QTF	$(CD)^2$
1234	72	5	0.1637	75	5	0.1636	68	0	0.1661	ABCE *	51	E	0.1636
12345	73	5	0.1637	74	5	0.1639	69	5	0.1661	ABCDE	52	D,E	0.1636

A comparison between the designs reveals that the newly constructed designs consistently exhibit fewer level changes (51, 52) than the corresponding standard designs, resulting in improved operational efficiency.

In the standard design as well as in newly constructed designs there are only two-fold-over plans that have all LTF main effects. Furthermore, in terms of QTF factors, the standard design has at most one QTF factor, whereas the newly constructed designs include fold-over plans with less than or equal to two QTF factors (e.g., the fold-over plan ABCDE).

It is important to note that some fold-over plans, despite having favorable properties in terms of level changes and number of LTF factors, may lack in terms of QTF factors. For instance, the fold-over plan 1234 in the standard 5-1.3 design has no QTF factors.

Table 5. Standard and new design for 2^{6-2}(n= 6 and p=2)

Standard Design 2^{6-2}										New design 2^{6-2}			
Foldover plan	6-2.1 1,2,3,4,5=123, 6=124			6-2.2 1,2,3,4, 5=12,6=134			6-2.3 1,2,3,4, 5=12, 6=34			A=1, B=3, C=4, D=23, E=BC, F=CD			
	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	Foldover plan	NLC	QTF	$(CD)^2$
1234	94	5,6	0.22	95	6	0.2225	72	0	0.2253	ABC	61	F	0.22252
12345	93	5,6	0.22	96	5,6	0.2198	73	5	0.2225	ABCE	62	E,F	0.22004
12346	93	5,6	0.22	94	6	0.2229	73	6	0.2225	ABCDE	62	D,E,F	0.2225
123456	92	5,6	0.2207	95	5,6	0.22	74	5,6	0.2197	ABCDF	61	D,F	0.22252
										ABCEF	61	E,F	0.22252
										ABCDEF	62	D,E,F	0.22005

Similarly, Table 5 presents selected fold-over plans for all standard 2^{6-2} designs provided by Li & Lin (2003) and fold-over plan for the newly constructed design respectively. A comparison of both the designs reveals that the newly constructed designs exhibit a lower NLC compared to the standard designs. While the standard designs include four foldover plans where all main effects are LTF, the newly constructed design provides six such plans. Furthermore, the newly constructed designs achieve up to three QTF factors (notably in the fold-over plan ABCDE and ABCDEF), whereas the standard designs attain only two at most. Among the standard designs, Design 6-2.3 has the fewest level changes. In contrast, the fold-over plans ABC, ABCDF and ABCEF from Table 5 exhibit the lowest NLC, although they may contain one or two QTF factors.

Table 6. Standard and new design for 2^{7-3}(n= 7 and p=3)

Standard Design 27-3																New Design 27-3			
Foldover plan	7-3.1 5 = 123,6 = 124,7=134			7-3.2 5 = 12,6=13,7 = 234			7-3.3 5 = 12,6 = 13,7 = 24			7-3.4 5 = 12,6=13,7 = 14			7-3.5 5 = 12,6 = 13,7 = 23			A=1, B=3, C=4, D=23, E=BC, F=CD, G=BCD			
	NLC	QTF	(CD)2	NLC	QTF	(CD)2	NLC	QTF	(CD)2	NLC	QTF	(CD)2	NLC	QTF	(CD)2	Fold-over plan	NLC	QTF	(CD)2
1234	121	5,6,7	0.2973	120	0	0.2935	104	0	0.297	120	0	0.2973	100	0	0.3004	ABC	73	F	0.2935
12345	120	5,6,7	0.2935	121	5	0.2904	105	5	0.2931	121	5	0.2935	101	5	0.2935	ABCD	74	D,F	0.2935
12346	120	5,6,7	0.2935	121	6	0.2904	105	6	0.2935	121	6	0.2935	101	6	0.2935	ABCE	74	E,F	0.2879
12347	120	5,6,7	0.2935	121	7	0.2942	105	7	0.2935	121	7	0.2935	101	7	0.2935	ABCDE	75	D,E,F	0.2934
123456	119	5,6,7	0.2903	122	5,6	0.2873	106	5,6	0.2904	122	5,6	0.2903	102	5,6	0.2935	ABCDF	73	D,F	0.2935
123457	119	5,6,7	0.2903	122	5,7	0.2879	106	5,7	0.2904	122	5,7	0.2903	102	5,7	0.2935	ABCG	74	E,F	0.2935
123467	119	5,6,7	0.2903	122	6,7	0.2904	106	6,7	0.29	122	6,7	0.2903	102	6,7	0.2935	ABCEF	73	F,G	0.2935
1234567	122	5,6,7	0.2879	123	5,6,7	0.2879	107	5,6,7	0.2876	123	5,6,7	0.2879	103	5,6,7	0.2879	ABCDEF	74	D,F,G	0.2935
																ABCEG	75	E,F,G	0.2935
																ABCDEG	76	D,E,F,G	0.2935
																ABCFG	73	F,G	0.2935
																ABCDFG	74	D,F,G	0.2935
																ABCEFG	74	E,F,G	0.2935
																ABCDEFG	75	D,E,F,G	0.2878

We observe standard designs 2^{7-3} have 2^3 (Table 6) and 2^{8-4} have 2^4 (Table 7(a)) fold-over plans that have all main effects LTF, in comparison the newly constructed designs have

$2^4 - 2$ (refer to Table 6) and $2^5 - 2$ (Table 7(b)) foldover plans respectively where all main effects are LTF.Thus we can generalize that for a standard 2^{n-p} design there are 2^p foldover plans with all main effects LTF and their corresponding new designs have $2^{p+1} - 2$ foldover plans with all main effects LTF. Additionally, the newly constructed designs have more QTF effects.

On comparison of both standard and newly constructed designs in terms of CD_2 discrepancy, it can be observed that the new designs exhibit same level of uniformity as the standard designs; change if any is almost negligible.

Table 7(a). Standard Design for 2^{8-4} (n= 8 and p=4)

Standard Design 2^{8-4}																		
	8-4.1			8-4.2			8-4.3			8-4.4			8-4.5			8-4.6		
	5 = 123,6 = 124,7 = 134,8 = 234			5 = 12,6 = 13,7 = 14,8 = 234			5 = 12,6 = 13,7 = 24,8 = 34			5 = 12,6=13,7 = 23, 8 =1234			5 = 12,6 = 13,7 = 23,8 =14			5 = 12,6 = 13,7 =23,8=123		
Foldover plan	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$	NLC	QTF	$(CD)^2$
1234	132	5,6,7,8	0.3691	131	8	0.3785	108	0	0.3829	120	8	0.3834	128	0	0.3864	123	8	0.3822
12345	131	5,6,7,8	0.3695	132	5,8	0.3751	109	5	0.3782	121	5,8	0.3751	129	5	0.3782	124	5,8	0.3785
12346	131	5,6,7,8	0.3695	132	6,8	0.3751	109	6	0.3782	121	6,8	0.3751	129	6	0.3782	124	6,8	0.3785
12347	131	5,6,7,8	0.3695	132	7,8	0.3751	109	7	0.3782	121	7,8	0.3751	129	7	0.3785	124	7,8	0.3785
12348	131	5,6,7,8	0.3695	130	8	0.3802	109	8	0.3782	121	8	0.3820	129	8	0.3820	122	8	0.3942
123456	130	5,6,7,8	0.3691	133	5,6,8	0.3716	110	5,6	0.3751	122	5,6,8	0.3747	130	5,6	0.3777	125	5,6,8	0.3820
123457	130	5,6,7,8	0.3691	133	5,7,8	0.3716	110	5,7	0.3751	122	5,7,8	0.3747	130	5,7	0.3782	125	5,7,8	0.3820
123458	130	5,6,7,8	0.3691	131	5,8	0.3751	110	5,8	0.3743	122	5,8	0.3747	130	5,8	0.3746	123	5,8	0.3820
123467	130	5,6,7,8	0.3691	133	6,7,8	0.3716	110	6,7	0.3743	122	6,7,8	0.3747	130	6,7	0.3782	125	6,7,8	0.3820
123468	130	5,6,7,8	0.3691	131	6,8	0.3751	110	6,8	0.3751	122	6,8	0.3747	130	6,8	0.3746	123	6,8	0.3820
123478	130	5,6,7,8	0.3691	131	7,8	0.3751	110	7,8	0.3751	122	7,8	0.3747	130	7,8	0.3743	123	7,8	0.3820
1234567	129	5,6,7,8	0.3695	134	5,6,7,8	0.3681	111	5,6,7	0.3712	123	5,6,7,8	0.3681	131	5,6,7	0.3716	126	5,6,7,8	0.3785
1234568	129	5,6,7,8	0.3691	132	5,6,8	0.3716	111	5,6,8	0.3712	123	5,6,8	0.3751	131	5,6,8	0.3751	124	5,6,8	0.3785
1234578	129	5,6,7,8	0.3695	132	5,7,8	0.3716	111	5,7,8	0.3712	123	5,7,8	0.3751	131	5,7,8	0.3746	124	5,7,8	0.3785
1234678	129	5,6,7,8	0.3695	132	6,7,8	0.3716	111	6,7,8	0.3712	123	6,7,8	0.3751	131	6,7,8	0.3746	124	6,7,8	0.3785
12345678	128	5,6,7,8	0.3723	133	5,6,7,8	0.3695	112	5,6,7,8	0.3687	124	5,6,7,8	0.3691	132	5,6,7,8	0.3687	125	5,6,7,8	0.3695

Table 7(b). New design for 2^{8-4} (n= 8 and p=4)

New design			
A=1, B=3, C=4, D=23, E=BC, F=CD, G=BCD, H=ABD			
Fold-over plan	NLC	QTF	$(CD)^2$
ABC	89	F	0.3751
ABCD	90	D,F	0.3747
ABCE	90	E,F	0.3691
ABCDE	91	D,E,F	0.3751
ABCDF	89	D,F	0.3751
ABCG	90	F,G	0.3747
ABCDG	91	D,F,G	0.3820
ABCH	90	F,H	0.3747
ABCDH	91	D,F,H	0.3751
ABCEF	89	E,F	0.3751

ABCDEF	90	D,F,H	0.3747
ABCEG	91	E,F,G	0.3751
ABCDEG	92	D,E,F,G	0.3747
ABCEH	92	E,F,G,H	0.3747
ABCDEH	93	D,E,F,G,H	0.3751
ABCFG	89	F,G	0.3751
ABCDFG	90	D,F,G	0.3747
ABCFH	89	F,H	0.3820
ABCDFH	90	D,F,H	0.3747
ABCGH	91	F,G,H	0.3751
ABCEFG	90	E,F,G	0.3747
ABCDEFG	91	D,E,F,G	0.3681
ABCEFH	90	E,F,H	0.3747
ABCDEFH	91	D,E,F,H	0.3751
ABCEGH	92	E,F,G,H	0.3747
ABCDEGH	93	D,E,F,G,H	0.3751
ABCFGH	90	F,G,H	0.3747
ABCDFGH	91	D,F,G,H	0.3751
ABCEFGH	91	E,F,G,H	0.3751
ABCDEFGH	92	D,E,F,G,H	0.3691

4.2 Comparison of Newly Constructed Designs with Cheng and Steinberg (1991) and Coster (1993) designs

Cheng and Steinberg (1991) proposed a simple yet effective algorithm for generating run orders of 2^{n-p} designs. Their method is a modification of earlier algorithms developed by Cheng (1985) and Coster & Cheng (1988), aimed at producing LTF run orders while maintaining nearly the maximum possible NLC.

As an example, consider a 2^4 factorial design with generators: $g_1 = abcd$ (the longest string in the design), $g_2 = abc$, $g_3 = abd$, and $g_4 = bcd$.

The construction begins with the identity run (1) and *abcd* as the first two runs. Reversing this sequence and multiplying by *abc* produces the third and fourth runs: *d* and *abc*. Continuing the process, the reversed order of the first four runs is multiplied by *abd*to generate the next eight runs, and finally, reversing this sequence and multiplying by *bcd* yields the complete run order:(1), abcd, d, abc, cd, ab, c, abd, ac, bd, acd, b, ad, bc, a, bcd.

This reverse fold-over algorithm typically results in designs where most, if not all, factors are LTF, and a significant number of factors are also QTF.

Table 8. 32 run design by Cheng & Steinberg (1991)

Design	Run sequence for 32 run design	NLC	LTF	QTF	$(CD)^2$
2^5	abcde, abcd, abce, ade, bde	137	All	B,D,E	0.1636
2^{6-1}	abcdef, abcd, abef, bcde, acef	156	All	A,C,D,E,F	0.2200
2^{7-2}	acdefg, bcdefg, abcde, abcfg, adg	177	All	A,D,F,G	0.2872
2^{8-3}	acdefgh, bcdefgh, abcde, abcfg, aef	201	All	A,E,F,G	0.3681

Table 8 presents 32-run designs constructed using method proposed by Cheng and Steinberg (1991). As compared to the design constructed by Cheng and Steinberg (1991), a notable reduction in the NLC can be observed in the newly constructed design (Table 4, 5, 6 and 7(b)). While both designs achieve an equal number of LTF factors but in some cases, Cheng and Steinberg's design include a greater number of QTF factors. The newly constructed designs offer a more efficient run order by significantly minimizing level changes. This makes it particularly advantageous in experimental settings where frequent level changes are costly or impractical.

Coster (1993) presents generator sequences that can be used in combination with Coster and Cheng's (1988) GFS to generate systematic run sequence that minimizes, a cost function equal to the number of factors level change in the run sequence while also having all factor main effect components orthogonal to a polynomial trend.

For example consider a 32 run design with run sequence: bcef, adef, cf, cd, bd i.e. z_1 = bcef, z_2=adef, z_3 = cf, z_4 =cd and z_5=bd. Then the sequence of generators to be used with GFS for the design are given by

$$\boldsymbol{g_i} = \left(\prod_{j=1}^{i-1} \boldsymbol{g_j^{s-1}}\right)\boldsymbol{z_i}$$

In our case s = 2, thus $g_1 = z_1$ = bcef, $g_2 = g_1z_2$ = abcd, $g_3 = (g_1g_2)z_3$ = acde, $g_4 = (g_1g_2g_3)z_4$ = df, and $g_5 = (g_1g_2g_3g_4)z_5$ = bc. After applying generalized fold-over algorithm to these generators we will get the run order of 32 run design; (1), bcef, abcd, adef, acde, abdf, be, cf, df, bcde, abcf, ae, acef, ab, bdef, cd, bc, ef, ad, abcdef, abde, acdf, ce, bf, bcdf, de, af, abce, abef, ac, cdef, bd. The design has 110 level changes with all main effects LTF and three QTF main effects are B, C, D.

A sufficient condition for trend free property to hold true when the run order is generated by GFS is mentioned in the form of a theorem by Coster (1993) quoted as:

"Theorem: For G generator, the run order is k-trend free if each factor appears at least (k + 1) times at a non-zero level in the generator sequence or, if a factor does not appear at least (k + 1) times, that factor is at a non-zero level at least once in some between block generator."

Table 9 shows 32-run designs constructed by Coster (1993) using the generalized fold-over algorithm along with generator sequence, QTF main effects and uniformity.

Table 9. 32 run design by Coster (1993)

Design	Run sequence for 32-run design	Generator(s)	NLC	QTF	$(CD)^2$
6-1.1	bcef, adef, cf, cd, bd	bcef, abcd, acde, df, bc	110	B,C,D	0.2197
6-1.2	de, af, bcef, cf, ad	de, adef, abce, be, acdf	70	A,D,E	0.2197
6-1.3	de, bf, ce, af, bd	de, bdef, bcef, acef, abdf	62	B,D,E,F	0.2224
7-2.1	abce, bcdfg, dg, fg, ce	abce, adefg, bcf, df, cefg	118	C,E,F	0.2872
7-2.2	beg, acg, cdef, dg,ab	beg, abce, adegf, cefg, abdg	94	A,B,E,G	0.2872
7-2.3	eg,df,ab,cg,bef	eg, defg, abdf, abcg, bcefg	63	B,E,F,G	0.2928
8-3.1	eh, abcd, efg, ach, abf	eh, abcdeh, abcdefg, acefgh, bcfh	85	A,B,C,E,F,H	0.3681
8-3.2	abcd, acfg, abefh, cdf, ace	abcd, bdfg, bcegh, abcdeh, adef	125	A,B,C,D,E	0.3681
8-3.3	eh, abcdfgh, bdh, cdf, efg	eh, abcdefg, acfg, bcfh, cdeg	109	C,E,F,G	0.3681

When comparing the newly constructed designs with those developed by Coster (1993) using the generalized fold-over algorithm, several observations can be made. The designs 6-1.1 and 6-1.2 exhibit a higher NLC (110 and 70) as well as a greater number of QTF factors (3, 3) than the corresponding newly constructed designs (Table 5) (minimum 61 and maximum 62 level changes and QTF factors 1 - 3). In the case of design 6-1.3, compared to the foldover plans ABC, ABCDF and ABCEF of the newly constructed 2^{6-2} design, the newly constructed designs show fewer level changes (61 vs. 62), but one or two QTF factor, whereas 6-1.3 includes four such factors.

For the 7-2 series**,** designs 7-2.1 and 7-2.2 have a greater NLC (118 and 94) than the newly constructed design (73 minimum and 76 maximum). However, design 7-2.3 shows fewer level changes (63) in comparison. Similarly, in the 8-3 series, designs 8-3.2 and 8-3.3 exhibit more level changes (125 and 109) than the newly constructed designs (89 minimum and 93 maximum), while design 8-3.1 has fewer level changes (85) as compared to newly constructed designs with minimum 89 level changes.

It is important to note that all the designs considered so far have all the main effects LTF. The fold-over scheme proposed in this paper is different from the reverse fold-over scheme of Cheng and Steinberg (1991) and GFS of Coster and Cheng (1988) as these schemes did not involve changing of the factors level.

5. Conclusion

This paper presents a new method of construction of 2^{n-p} FFDs, for $5 \leq n \leq 15$; $1 \leq p \leq 11$. These designs achieve lesser number of level changes and have all main effects linear trend free as compared to the standard design of Li & Lin (2003).

The comparative analysis of reverse fold-over, and newly constructed designs shows that while Cheng and Steinberg (1991) proposed designs which have maximum number of level changes, almost all factors LTF and few factors QTF, the newly constructed designs stand out for achieving all linear trend-free factors with a significantly lower NLC, making them more practical for real-world implementation where frequent level changes may be costly or logistically challenging.

In comparison with designs generated using Coster (1993) generalized fold-over algorithm, the newly constructed designs generally have fewer level changes, though they may have smaller number of quadratic trend-free factors in some cases. Overall, the new designs provide an effective alternative for practitioners seeking to minimize level changes while maintaining strong trend-free properties in factorial experiments.

On comparison with Cheng and Steinberg (1991) and Coster (1993) foldover designs in terms of CD_2 discrepancy, it can be observed that the new designs and standard designs exhibit almost same level of uniformity; change if any is close to negligible.

The newly constructed designs are foldover designs that have significantly lower number of level changes, all main effects linear trend free and some main effects quadratic trend free.

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Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: GUPTA, S., BUDHRAJA, V.**; Data curation:** GUPTA, S., BUDHRAJA, V., SHARMA, S. **; Formal analysis:** GUPTA, S., BUDHRAJA, V., SHARMA, S. **; Investigation:** GUPTA, S., BUDHRAJA, V., SHARMA, S. **; Methodology**: GUPTA, S., BUDHRAJA, V., SHARMA, S.**; Software:** GUPTA, S., BUDHRAJA, V., SHARMA, S**.; Supervision**: GUPTA, S., BUDHRAJA, V.**;Validation**: GUPTA, S., BUDHRAJA, V., SHARMA, S.**; Visualization:** GUPTA, S., BUDHRAJA, V., SHARMA, S.; **Writing - original draft:** GUPTA, S., BUDHRAJA, V., SHARMA, S. **; Writing - review and editing:** GUPTA, S., BUDHRAJA, V., SHARMA, S**.**

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Appendix : R code for the construction of new designs

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Design<-{
 levels <- c(-1, 1)
 design <- expand.grid(F1 = levels, F2 = levels, F3 = levels, F4 = levels)
 # Step 2: Create 2-factor interactions (multiply columns)
 design$F1F2 <- design$F1 * design$F2
 design$F1F3 <- design$F1 * design$F3
 design$F1F4 <- design$F1 * design$F4
 design$F2F3 <- design$F2 * design$F3
 design$F2F4 <- design$F2 * design$F4
 design$F3F4 <- design$F3 * design$F4
count_changes<- function(x) {
  sum(x[-1] != x[-length(x)])}
 # Calculate level changes for all columns
change_list<- sapply(design, count_changes)
 #  Create interaction (23)
 design$F23 <- design$F2 * design$F3
 # Step 3: Select independent columns:
 # F1 → A, F3 → B, F4 → C, F23 → D
indep<- data.frame(
  A = design$F1,
  B = design$F3,
  C = design$F4,
  D = design$F23)
count_changes<- function(x) {
  sum(x[-1] != x[-length(x)])}
generate_design<- function(indep_cols, p) {
  # Rename independent columns as A,B,C,D,...
  names(indep_cols) <- LETTERS[1:ncol(indep_cols)]
  # Helper
count_changes<- function(x) sum(x[-1] != x[-length(x)])
  # ----- All interactions -----
  int2 <- list()
  int3 <- list()
  int4 <- list()
  # 2-factor interactions
fac<- names(indep_cols)
for(i in 1:(length(fac)-1)) {
for(j in (i+1):length(fac)) {
      nm <- paste0(fac[i], fac[j])
      int2[[nm]] <- indep_cols[[fac[i]]] * indep_cols[[fac[j]]]}}
  # 3-factor interactions
for(i in 1:(length(fac)-2)) {
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for(j in (i+1):(length(fac)-1)) {
for(k in (j+1):length(fac)) {
        nm <- paste0(fac[i], fac[j], fac[k])
        int3[[nm]] <- indep_cols[[fac[i]]] *
indep_cols[[fac[j]]] *
indep_cols[[fac[k]]]}}}
   # 4-factor (full interaction)
   if (length(fac) == 4) {
     nm <- paste(fac, collapse = "")
     int4[[nm]] <- indep_cols[[fac[1]]] *
indep_cols[[fac[2]]] *
indep_cols[[fac[3]]] *
indep_cols[[fac[4]]]}
   # Combine all
all_int<- c(int2, int3, int4)
   # Compute level changes
   lc <- sapply(all_int, count_changes)
   # Pick p minimum-change interactions
   sorted <- sort(lc)
selected_int<- names(sorted)[1:p]
   # Extract selected columns
selected_cols<- lapply(selected_int, function(x) all_int[[x]])
   # Assign new factor names (E, F, G,…)
new_names<- LETTERS[(ncol(indep_cols)+1):(ncol(indep_cols)+p)]
for(i in 1:p) indep_cols[[new_names[i]]] <- selected_cols[[i]]
   # Return results
list(
selected_interactions = selected_int,
generated_design = indep_cols )}
  repeat {
p_input<- readline("Enter the value of p (1 to 11) to construct 2^(n-p) design: ")
   if (p_input == "") next   # ignore empty reads
p_val<- suppressWarnings(as.integer(p_input))
   if (!is.na(p_val) &&p_val>= 1 &&p_val<= 11) {
     break
   } else {
cat("Invalid input! p must be between 1 and 11.\n") }}
  result <- generate_design(indep, p_val)
  v<-result$selected_interactions
  design <- result$generated_design
  n <- ncol(design);n
  p <- length(v);p
  {cat("\n=====  INITIAL 16 RUN OF 2^(",n, "-", p, ") DESIGN =====\n", sep = "" )
   print(design)}
result$selected_interactions
result$generated_design
```

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count_changes<- function(x) sum(x[-1] != x[-length(x)])
 # ==== FINAL DESIGN FROM OUR FRACTIONAL FUNCTION ====
fold_over<- function(design, flip_factors) {
   folded <- design
   # Flip sign for selected factors
folded[, flip_factors] <- -1 * folded[, flip_factors]
   # Combine original + folded
full_design<- rbind(design, folded)
return(list(
folded_only = folded,
full_design = full_design))}
 design <- result$generated_design   # Final design
 # Show available factors
cat("Available factors:\n")
 print(colnames(design))
 repeat {
flip_input<- readline(
    "Enter factors to fold-over (comma separated, e.g. A,B,C,...): ")
   # Convert input to vector
flip_factors<- trimws(unlist(strsplit(flip_input, ",")))
   # Validation
   if (all(flip_factors %in% colnames(design)) && length(flip_factors) > 0) {
     break
   } else {
cat("❌ Invalid factor name(s). Please choose from:\n")
    print(colnames(design))}}
 # Apply fold-over
 out <- fold_over(design, flip_factors = flip_factors)
 # Outputs
 a<-out$folded_only
 {cat( "\n=====  FOLDOVER 16 RUNS OF 2^(", n, "-", p, ") DESIGN =====\n",sep = "")
  print(a)}
 Pause <- function(msg = "Press [enter] to continue...") {
readline(prompt = msg)}
Pause()
full_foldover<- out$full_design
cat( "\n===== FINAL 32 RUNS OF 16 RUN FOLDOVER 2^(",n, "-", p,") DESIGN =====\n", sep
= "" )
 print(full_foldover)
design <- out$full_design   # final design
max_replication<- 1   # allowed copies per run
# max_replication = 1 → no duplication allowed
# Count occurrences of each run
run_freq<- table(as.data.frame(design))
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# Identify runs exceeding allowed replication
violations <- run_freq[run_freq>max_replication]
if (length(violations) > 0) {
cat("❌ There is run duplication for the following runs:\n")
stop("Execution stopped due to run duplication in 32 run design.
Please try different foldover plan.")}
cat("✅ There is no run duplication. Proceeding with further computation.\n")
 pause <- function(msg = "Press [enter] to continue...") {
readline(prompt = msg) }
pause()
level_changes<- sapply(full_foldover, count_changes)
cat(
   "\n===== NUMBER OF LEVEL CHANGES FOR FINAL 2^(", n, "-", p,") DESIGN =====\n",
sep = "" )
  print(level_changes)
pause <- function(msg = "Press [enter] to continue...") {
readline(prompt = msg)}
pause()
cat("\n===== TOTAL NUMBER OF LEVEL CHANGES FOR FINAL 2^(",n, "-", p, ") DESIGN
=====\n",sep = "")
print(sum(level_changes ))
  # Time index
  t <- 1:nrow(full_foldover)
time_count<- function(factor_col, t) sum(factor_col * t)
  # Time count for each factor
time_counts_fold<- sapply(full_foldover, time_count, t = t)
cat("\n=== TIME COUNT VALUE FOR ALL FACTORS FOR FINAL DESIGN ===\n")
  print(time_counts_fold)
  #central descrepancy
full_foldover[full_foldover == -1] <- 0
  x<-as.matrix(full_foldover);x
  u<-as.matrix((x+0.5)/2);u
  n<-ncol(u);n
  N<-nrow(u);N
  CD2_phi<-function(u,N,n){
    term1<-(13/12)^n
    sum1<-0
for(k in 1:N){
    product1<-1
for(j in 1:n){
    product1<-product1*(1+(0.5*abs(u[k,j]-0.5))-(0.5*(abs(u[k,j]-0.5))^2))}
    sum1<-sum1+product1}
    sum3<-0
for(k in 1:N){
    sum2<-0
```

```
for(l in 1:N){
   product2<-1
for(i in 1:n){
 product2<-product2*(1+(0.5*abs(u[k,i]-0.5))+(0.5*abs(u[l,i]-0.5))-(0.5*abs(u[k,i]-
                                                                     u[l,i])))
    }
       sum2<-sum2+product2}
     sum3<-sum3+sum2}
   term2<-(2/N)*sum1
   term3<-(1/(N^2))*sum3
   t<-term1-term2+term3}
  b<-unname(CD2_phi(u,N,n))
cat("\n===== CENTRAL DESCREPANCY  (CD2)=====\n")
  print(b)}
```